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EECS 210 Applied Electromagnetic Theory

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Solution Midterm Exam

October 24, 2006

Open Book, Open Notes, Open Homework
Write directly on this exam

Sign Your Name:______________________________

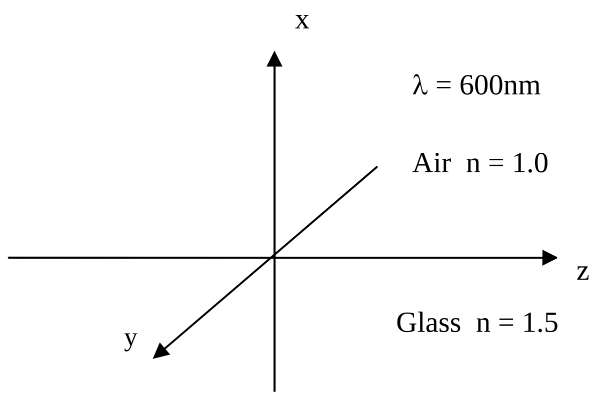
Print Your Name:______________________________

Problem	Possible	Score
I	50	
II	50	
III	50	
Total	150	

I. (50 Points) Plane Waves:

$$E_x = 0.5E_0 e^{i\frac{2\pi n_G}{\lambda}\left(0.3x+k_y y+0.6z\right)}$$

$$E_y = 1.0E_0 e^{i\frac{2\pi n_G}{\lambda}\left(0.3x+k_y y+0.6z\right)}$$



a) (10 Points) Find the angle that this wave makes with the y- axis **in the glass**.

$$k_y = \sqrt{1-(0.3)^2-(0.6)^2} = 0.742 \Rightarrow 42^0$$

b) (10 points) Write out the full (x,y,z) plane wave behavior of the transmitted field. (Leave the phasor amplitude and relative phase as an unknown).

ky and kz are continuous = 0.742(1.5) and 0.6(1.5) = 1.113 and 0.9

$$k_x = \sqrt{1-(1.113)^2-(0.9)^2} = \sqrt{-1.049} = i1.024$$

$$e^{1.024x+i(1.113y+0.9z)}$$

c) (20 Points) Find all **six** components of the vectors E and H traveling in the upward direction **inside the glass**.

$$\nabla \cdot E = 0 \Rightarrow (0.5)(0.3)+(1.0)(0.743)+E_z(0.6) = 0 \Rightarrow E_z = -1.49$$

$$H = \frac{1}{i\omega\mu}\nabla \times E$$

$$H = \frac{1}{i\omega\mu}\frac{2\pi n_G}{\lambda}E_0[\{0.742(-1.49)E_0 - 0.6(1.0)E_0\}\hat{x} + \{...\}\hat{y} + \{0.3(1.0) - 0.742(0.5)\}\hat{z}]$$

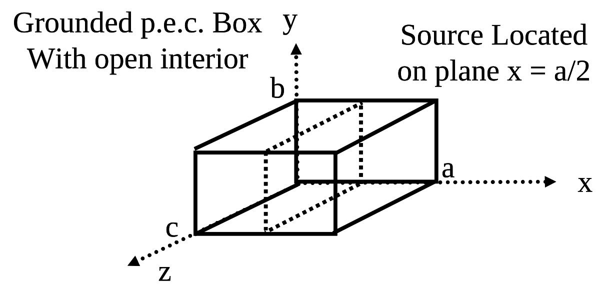
Then consolidate constant and vector terms.

c) (10 Points) Evaluate the Poynting vector component in the y-direction due to waves traveling in the +y-direction **in the glass**.

$$P_y = E \times H = E_z H_y - E_x H_z$$

Then plug in values from above.

II. (50 Points) Boundary Value Problem:



a) (15 points) Find the potential inside the inside the box when the potential on the plane x = a/2 is given by $\Phi(y,z)|_{x=a/2} = F\sin(\frac{3\pi y}{b})\sin(\frac{2\pi z}{c})$

$$\Phi(x,y,z)|_{x<a/2} = \sum_{l,m,n} A_{l,m}\sinh(\gamma_{m,n}x)\sin\left(\frac{m\pi y}{b}\right)\sin\left(\frac{n\pi z}{c}\right)$$

$$\Phi(x,y,z)|_{x>a/2} = \sum_{l,m,n} B_{l,m}\sinh[\gamma_{m,n}(a-x)]\sin\left(\frac{m\pi y}{b}\right)\sin\left(\frac{n\pi z}{c}\right)$$

$$\gamma_{m,n} = [\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{c}\right)^2]^{1/2}$$

$$A_{l,m}\sinh(\gamma_{m,n}a/2) = B_{l,m}\sinh[\gamma_{m,n}(a-x)] = F$$

b) (15 points) Find the charge on the plane x= a/2 associated with this potential.

$$D_x(x = a^+/2) - D_x(x = a^-/2) = \sigma(y,z)$$

$$D_x = \varepsilon E_x = -\varepsilon\nabla\Phi\cdot\hat{x} = -\varepsilon\frac{\partial\Phi}{\partial x}$$

Plug in field F from above and use derivative of sinh = gamma times cosh.
Both sides of a/2 contribute equally.

c) (20 Points) Write one sentence that names and outlines the methodology for each of the possible ways that could be used to solve for the potential produced by a charge distribution inside the box above.

Maximum of 20 points
N-1 expansion in 3 directions plus a required superposition to integrate in 3rd dimension across the charge cloud.(5+3 points)
N expansion (no integration needed) (4 points)
Integral representation with Dirichlet BC and then integrate over charge cloud. (4 points)
Image charge with 3d array of charges. (4 points)

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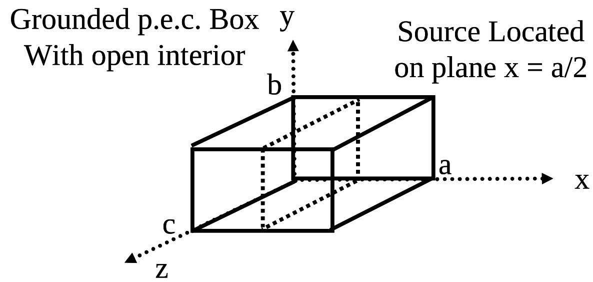
III. (50 Points) Integral Representation



a) (12 Points) Use the concept of a general Green's function to write an integral equation for the potential at a point (x1, y1, z1) where x1 < a/2.

$$\Phi(x_1, y_1, z_1) = \int_{V_2} \rho G(\bar{x}_1, \bar{x}_2) d^3x_2 + \int_{\partial V} \left(G(\bar{x}_1, \bar{x}_2) \frac{\partial \Phi}{\partial n_2} - \Phi \frac{\partial G(\bar{x}_1, \bar{x}_2)}{\partial n_2} \right) d^2x'$$

Here the volume has x range 0 to **ONLY** a/2 and, y range 0 to b and z ranges 0 to c.

b) (12 Points) Specify the boundary conditions on the Green's function such that **only the potential on the plane a/2 is needed** to find the potential at a point (x1, y1, z1) where x1 < a/2.

G zero on both x = 0 and **a/2**, and G = 0 on b = 0 and b, and G = 0 on z = 0 and c.

c) (12 Points) Write down an eigenfunction expansion for this Green's Function.

$$\Phi(x, y, z) = \frac{\sum_{l,m,n} A_{n,lm} \sin\left(\frac{l\pi x}{a/2}\right) \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{n\pi z}{c}\right)}{\left(\frac{l^2}{(a/2)^2} + \frac{n^2}{b^2} + \frac{m^2}{c^2}\right)}$$

d) (14 Points) Describe how the integral representation in a) could be converted into an integral equations to find the charge on the walls for x < a/2.

Set ρ = 0 inside the half-volume.
Change Green's function to at least have normal derivative zero on x = a/2 plane.
Substitute $\frac{\partial \Phi}{\partial n}|_{x=a/2} = -\frac{\sigma}{\varepsilon}$ Then expand σ as function of y and z in N charge patches σ_n.
Then take limit of integral representation as (x,y,z) approaches (a/2, y, z) in center of each patch to get N equations. Put in matrix form and solve for the charges σ_n on the N patches