1 Jacobian Warm-Up

Consider the following function $f : \mathbb{R}^2 \mapsto \mathbb{R}^3$

$$f(x_1, x_2) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \\ f_3(x_1, x_2) \end{bmatrix} = \begin{bmatrix} x_1^2 - x_2^2 \\ x_1^2 + x_1x_2^2 \\ x_1 \end{bmatrix}$$

Calculate its Jacobian.

2 Linearization

Consider a mass attached to two springs:

![Diagram of a mass connected to two springs](image)

We assume that each spring is linear with spring constant $k$ and resting length $X_0$. We want to build a state space model that describes how the displacement $y$ of the mass from the spring base evolves. The differential equation modeling this system is

$$\frac{d^2y}{dt^2} = -\frac{2k}{m}(y - X_0 \frac{y}{\sqrt{y^2 + \theta^2}}).$$

a) Write this model in state space form $\dot{x} = f(x)$.  

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b) Find the equilibrium of the state-space model. You can assume $X_0 < a$.

c) Linearize your model about the equilibrium.

3 Discretization

Consider a cart of mass $M$, pushed with a force $u(t)$ with position, $p(t)$, and velocity, $v(t)$. Hence, we have:

$$\frac{d}{dt} p(t) = v(t)$$

$$\frac{d}{dt} v(t) = \frac{u(t)}{M}$$

We will apply a constant input between any time $t \in [t, t + T]$. Here $T$ is our time step.

Find a discretized system of equations for this system.