EECS 16B  Designing Information Devices and Systems II
Spring 2018  J. Roychowdhury and M. Maharbiz  Homework 2

This homework is due on Thursday, February 8, 2018, at 11:59AM (NOON).
Self-grades are due on Monday, February 12, 2018, at 11:59AM (NOON).

Pre-Lab

1. Mystery Microphone

You are working for APPLE Microphone Corporation when your manager asks you to test a batch of the company’s new microphones. You grab one of the new microphones off the shelf, play a uniform tone with varying frequencies, and measure the resultant peak-to-peak voltages using an oscilloscope. In order to play a uniform tone, you use a tone generator which outputs an audio wave of uniform amplitude for all frequencies involved. Below is the data obtained from your experiments:

<table>
<thead>
<tr>
<th>Input frequency (Hz)</th>
<th>Output peak-to-peak (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.4</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
</tr>
<tr>
<td>40</td>
<td>0.5</td>
</tr>
<tr>
<td>60</td>
<td>0.6</td>
</tr>
<tr>
<td>100</td>
<td>2.2</td>
</tr>
<tr>
<td>160</td>
<td>2.3</td>
</tr>
<tr>
<td>320</td>
<td>5</td>
</tr>
<tr>
<td>640</td>
<td>5</td>
</tr>
<tr>
<td>1200</td>
<td>4.8</td>
</tr>
<tr>
<td>2500</td>
<td>4.7</td>
</tr>
<tr>
<td>5000</td>
<td>4.6</td>
</tr>
<tr>
<td>10000</td>
<td>1.6</td>
</tr>
<tr>
<td>12000</td>
<td>1.5</td>
</tr>
<tr>
<td>16000</td>
<td>1.4</td>
</tr>
<tr>
<td>20000</td>
<td>1.3</td>
</tr>
</tbody>
</table>

(a) Plot the output peak-to-peak voltage against the input frequency in log scale.

(b) What do you notice? To what frequencies is the microphone most sensitive, and to what frequencies is the microphone least sensitive?

You report these findings to your manager, who thanks you for the preliminary data and proceeds to co-ordinate some human listener tests. In the meantime, your manager asks you to predict the effects of the microphone recordings on human listeners, and encourages you to start thinking more deeply about the relationships.

1 Advanced Powerful Pleasant Lovely Experiences Microphone Corporation
(c) For testing purposes, you have a song with sub-bass (150 Hz or less), mid-range (≈ 1 kHz), and some high frequency electronic parts (> 12 kHz). Which frequency ranges of the song would you be able to hear easily, and which parts would you have trouble hearing? Why?

(d) After a few weeks, your manager reports back to you on the findings. Apparently, this microphone causes some people’s voices to sound really weird, resulting in users threatening to switch to products from a competing microphone company. It turns out that we can design some filters to “fix” the frequency response so that the different frequencies can be recorded more equally, thus avoiding distortion. Imagine that you have a few (say up to 4 or so) blocks. Each of these blocks detects a set range of frequencies, and if the signal is within this range, it will switch on a op-amp circuit of your choice. For example, it can be configured to switch on an op-amp filter to double the voltage for signals between 100 Hz and 200 Hz. What ranges of signals would require such a block, and what gain would you apply to each block such that the resulting peak-to-peak voltage is about 5 V for all frequencies?

2. Analyzing Mic Board Circuit

In the third lab, we will use a micboard circuit below to convert acoustic signal into an electrical signal. The microphone (MIK1) can be modeled as a signal-dependent current source, $I_{MIK1} = k \sin(\omega t)$, where $I_{MIK1}$ is the current flowing from VDD to VSS, $k$ is the audio to current conversion ratio, and $\omega$ is the signal’s frequency (in rad/s). A voltage buffer follows the microphone circuit to prevent it from experiencing any impedance loading effects from the next stage circuitry. $R_2$ is the feedback resistor that is placed to cancel the input current offset caused by op-amp imperfections. $\text{Ignore } R_2 \text{ in your analysis as it is beyond the scope of this course.}$

$C_1$ is a very large capacitor that eliminates any DC voltage coming from the first stage. $\text{Hint: You can treat } C_1 \text{ such that it passes only AC signals. No DC signals pass through } C_1.$

$R_6$ is a very large resistor that decreases the effect from AMP2. $\text{Hint: You can ignore } R_6 \text{ in your analysis.}$ For example, if $R_6$ is 0 $\Omega$, then $V_3$ would be just a DC signal.

Finally, the variable gain amplifier amplifies the signal to the desired amplitude. Note that $R_{51}$ and $R_{52}$ are resistors of the potentiometer ($R_5 = R_{51} + R_{52}$). $V_{OS}$ denotes the DC voltage source that we apply to the circuit. The goal of this circuit is to maximize $V_{out}$ amplitude which must be from 0 V to 3.3 V while simultaneously setting its DC offset to 1.65 V. VDD and VSS are 5 V and $-5$ V, respectively.
Figure 1: Mic Board Schematic

(a) First, consider the case where $V_{OS} = 0\ V$, where there is no DC offset, $R_3 = \infty$, and $R_4 = 0$. Describe $V_1$, $V_2$, $V_3$, and $V_{out}$ in terms of $k$, $\omega$, $t$, $R_1$, $R_{52}$, and $R_5$.

(b) What component values for $R_{51}$ and $R_{52}$ maximize $V_{out}$ amplitude without clipping from VDD or VSS at any node in the circuit? In this case, what is the gain of the variable gain amplifier? Set $k = 10^{-5}$, $R_1 = 10\ k\Omega$, $R_3 = \infty$, $R_4 = 0\ \Omega$, $V_{OS} = 0\ V$, and $R_5 = 50\ k\Omega$. Show your work.

**ADC Input Range Requirements:** The ADC after the Micboard circuit has an input range from 0\ V to 3.3\ V, i.e., any signal out of this range will be clipped. Therefore, we want $V_{out}$ to be from 0\ V to 3.3\ V and centered at 1.65\ V.

(c) Describe how you can center the signal at $V_3$ on the micboard at 1.65\ V. Which components out of $C_1$, $R_1$, $R_2$, $R_3$, $R_4$, $R_{51}$, and $R_{52}$ will you need to specify to do this and what values will you assign them? There may be multiple component values satisfying the above constraints. (Note: Approximately 1.65\ V will suffice.)

(d) Express $V_{out}$ in terms of $k$, $\omega$, $t$, $R_1$, $R_3$, $R_4$, $R_{52}$, $R_5$, and $V_{OS}$. Provide values for $R_3$, $R_4$, $R_{52}$, and $V_{OS}$ that maximize the signal amplitude and satisfies the ADC input range requirements specified in the previous part. Set $k = 10^{-5}$, $R_1 = 10\ k\Omega$ and $R_5 = 50\ k\Omega$. There may be multiple component values satisfying the above constraints. Show your work. (Hint: Use superposition.)
### Problems

#### 3. Two Inductors

Consider the circuit below. Assume that for \( t < 0 \), the circuit has reached steady state (\( V_1 = 0, V_2 = 0 \)). At \( t = 0 \), the switch connected to \( V_s \) closes. Assume that \( V_s = 10 \text{V}, R_1 = R_2 = 5 \text{k}\Omega, \) and \( L_1 = L_2 = 0.2 \text{H} \).

![Two Inductor Circuit with Voltage Source](image)

Figure 2: Two Inductor Circuit with Voltage Source

(a) First, use Kirchoff’s Laws and the inductor equation \( V = L \frac{dI}{dt} \) to find the second order differential equation for this system in terms of \( I_2(t), L_1, L_2, R_1, \) and \( R_2 \).

(b) Now cast this second order differential equation into the following form:

\[
\frac{d\vec{i}}{dt} = A\vec{i},
\]

where

\[
\vec{i} = \begin{bmatrix} I_2(t) \\ \frac{dI_2(t)}{dt} \end{bmatrix}.
\]

Plug in values to get a numerical matrix.

(c) Find the eigenvalues of \( A \). Are they real or complex?

(d) Using the initial conditions, what is the solution to the differential equation?

(e) Find \( I_1(t) \).

(f) Sketch the current vs. time plots of \( I_1(t) \) and \( I_2(t) \).

#### 4. General RLC Responses

Consider the following circuit assume this circuit has reached steady state for \( t < 0 \):

![General RLC Responses](image)

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(a) Find the differential equation in terms of $V_c$ that describes this circuit for $t \geq 0$ and solve it in terms of $V_s$, $L$, $R$, and $C$. Make sure to consider all three cases.

(b) At what frequency is this circuit going to oscillate? Your answer should be in terms of $R$.

(c) Roughly sketch the shape of the transient response of $V_c(t)$ for $t \geq 0$ when $R = 1 \Omega$.

(d) Roughly sketch the shape of the transient response of $V_c(t)$ for $t \geq 0$ when $R = 10 \Omega$.

(e) Roughly sketch the shape of the transient response of $V_c(t)$ for $t \geq 0$ when $R = 1 \text{k}\Omega$.

5. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing AC circuits (circuits driven by sinusoidal inputs) in the phasor domain. In this problem, we will walk you through the steps with a concrete example. Consider the circuit below.

The components in this circuit are given by:

Voltage source:

$$v(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

Resistors:

$$R_1 = 5 \Omega, \quad R_2 = 5 \Omega, \quad R_3 = 1 \Omega$$

Inductors:

$$L_1 = 50\text{mH}, \quad L_2 = 20\text{mH}$$

Capacitor:

$$C_1 = 2\text{mF}$$

(a) To begin with, transform the given circuit to the phasor domain.

(b) Write out KCL for node $N_1$ and $N_2$ in the phasor domain in terms of the currents provided.

(c) Find expressions for each current in terms of node voltages in the phasor domain. The node voltages $\tilde{V}_1$ and $\tilde{V}_2$ are the voltage drops from $N_1$ and $N_2$ to the ground.

(d) Write the equations you derived in part (b) and (c) in a matrix form, i.e., $A \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \tilde{b}$. Write out $A$ and $\tilde{b}$ numerically.
(e) Solve the systems of linear equations you derived in part (d) with any method you prefer and then find \( i_c(t) \).

6. (OPTIONAL) Complex Numbers

A common way to visualize complex numbers is to use the complex plane. Recall that a complex number \( z \) is often represented in Cartesian form.

\[
z = x + jy \text{ with } \text{Re}\{z\} = x \text{ and } \text{Im}\{z\} = y
\]

See the Figure 3 for how \( z \) looks like in the complex plane.

![Complex Plane](image)

Figure 3: Complex Plane

In this question, we will derive the polar form of a complex number and use this form to make some interesting conclusions.

(a) Calculate the length of \( z \) in terms of \( x \) and \( y \) as shown in Figure 3. This is the magnitude of a complex number and is denoted \(|z|\) or \( r \). *Hint.* Use the Pythagoras theorem.

(b) Represent the real and imaginary parts of \( z \) in terms of \( r \) and \( \theta \).

(c) Substitute for \( x \) and \( y \) in \( z \). Use Euler’s formula to conclude that,

\[
z = re^{j\theta}
\]

(d) In the complex plane, draw out all the complex numbers such that \(|z| = 1\). What are the \( z \) values where the figure intersects the real axis and the imaginary axis?

(e) If \( z = re^{j\theta} \), prove that \( z^* = re^{-j\theta} \). Recall that the complex conjugate of a complex number \( z = x + jy \) is \( z^* = x - jy \).
(f) Show that,

\[ r^2 = z z^* \]

(g) Intuitively argue that,

\[ \sum_{k=0}^{2} e^{j2\pi k} = 0 \]

Do so by drawing out the different values of the sum and making an argument based on the vector sum.

7. **OPTIONAL** RLC Circuit

Now consider the circuit shown below:

(a) Assuming the circuit reaches steady state for \( t < 0 \), find the differential equation for \( V_{out} \) for \( t \geq 0 \)

(b) What are the initial conditions at \( t = 0 \) for this differential equation?

(c) Solve the differential equation. Consider all cases (underdamped, critically damped, overdamped).

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