This homework is due Monday January 25, 2016, at Noon.

1. **Homework process and study group**

   Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework?

2. **Circuits and Gaussian Elimination**

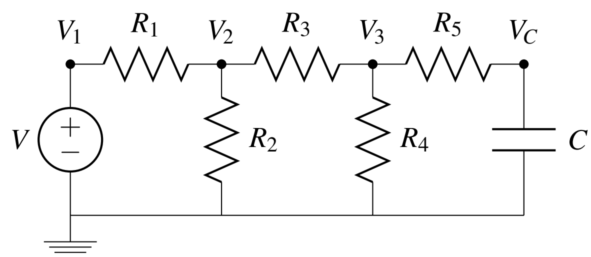

Figure 1: Example Circuit

   (a) Find a system of linear equations that could be solved to find the node voltages.
   (b) Given that the component values are $R_1 = 500\,\Omega$, $R_2 = 3\,\mathrm{k}\Omega$, $R_3 = 1\,\mathrm{k}\Omega$, $R_4 = 2\,\mathrm{k}\Omega$, and $R_5 = 4\,\mathrm{k}\Omega$, Solve the circuit equations using Gaussian elimination.
   (c) What's the voltage V_C across the capacitor?
   (d) How would you check your work? Do so.

3. **Solving Recurrence Relations**

   For this problem, we'll work with a sequence defined by the following recurrence relation, where $S[n]$ is the nth number in the sequence:

$$S[n+1] = 3S[n] - 2S[n-1]$$
$$S[0] = 0$$
$$S[1] = 1$$

   You can probably see how this could be computed recursively or iteratively, but let's try a linear-algebraic approach and see where it takes us.

   (a) Starting from the definition of the sequence, determine the matrix A to calculate $S[n+1]$ such that:

$$A \begin{pmatrix} S[n] \\ S[n-1] \end{pmatrix} = \begin{pmatrix} S[n+1] \\ S[n] \end{pmatrix}$$

(b) Find the eigenvalues λ_+ and λ_- of the matrix A.

(c) Find the eigenvectors associated with λ_+ and λ_- from above.

(d) How would you check your work? Do so. (Hint: Based on the eigenvalues you determined, how should this sequence behave for different initial conditions? Does it?)

(e) Using your previous results, diagonalize A.

(f) How can you use this new information to more efficiently compute any arbitrary $S[n]$ *without using any iteration or recursion*? (Hint: if a matrix $M = PDP^{-1}$ where D is a diagonal matrix, then think about $M^2 = MM$, M^3, and even M^n.)

(g) Finally, using your results from above, derive a closed-form expression with no summations, no recursion, and no matrix multiplications for $S[n]$.

4. Show it

(a) Suppose $\lambda_1, \ldots, \lambda_m$ are distinct eigenvalues of a matrix T and $\vec{v}_1, \ldots, \vec{v}_m$ are the corresponding eigenvectors. Show that $\vec{v}_1, \ldots, \vec{v}_m$ must be linearly independent.

(b) Show that if a vector $\vec{x}$ is in the null space of a matrix A, $\vec{x}$ must be orthogonal to all vectors in the column space of A^*. (Hint: does $\vec{x}$ have to be orthogonal to the columns of A^*? Remember, the $*$ means conjugate-transpose.)

5. Getting Rich (or not) with Linear Regression

In this problem, we will use the linear least-squares models we learned previously for stock market analysis, and find the best model to do so. This problem is meant to brush up your skills with Linear Regression as well as IPython numpy, which we will use extensively throughout the course.

Use the provided ipython notebook file accompanying this homework.

(a) How many weeks of data have we given to you in `nasdaq1.csv`? Plot it.

(b) What are the coefficients of the linear regression model for the data in the previous part? And what is the norm of the residual error? Plot the straight-line model together with the data.

(c) `nasdaq5.csv` contains similar data, but since January 2010. Plot the NASDAQ composite index over the past 5 years along with the prediction of your model and calculate the norm of the error vector. What is the norm of the error vector for 5 years of stock data when predicted using the linear regression model found using `nasdaq1.csv`?

(d) Repeat parts b) and c), but use a quadratic fitting model instead of a linear one. Use the 86 weeks of NASDAQ data to produce a model, find the coefficients for the quadratic regression, and extrapolate the model to 5 years of data. What is the norm of the error vector for both the 5 years of stock data and the 86 weeks of stock data? Plot the quadratic model together with the data for both 86 weeks and 5 years.

(e) Repeat parts b) and c) again, but this time use an exponential fitting model instead of a linear one. Using the 86 weeks of data, find the coefficients for the exponential regression, and extrapolate the model to 5 years of data. What is the norm of the error vector for both the 5 years of stock data and the 86 weeks of stock data? Plot the exponential model together with the data for both 86 weeks and 5 years.

(f) Which model had the lowest error? Why do you think that is?

(g) Can these models be used to predict future behavior? Why or why not?

6. **Your Own Problem**

Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student must submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?