1 Complex Numbers

A complex number \( z \) is an ordered pair \((x, y)\), where \( x \) and \( y \) are real numbers, written as \( z = x + jy \) where \( j = \sqrt{-1} \). A complex number can also be written in polar form as follows:

\[
z = |z| e^{j\theta}
\]

The magnitude \( z \) is denoted as \(|z|\) and is given by

\[
|z| = \sqrt{x^2 + y^2}.
\]

The phase or argument of a complex number is denoted as \( \theta \) and is given by

\[
\theta = \text{atan2}(y, x).
\]

Here, \( \text{atan2}(y, x) \) is a function that returns the angle from the positive x-axis to the vector from the origin to the point \((x, y)\).\(^{1}\)

The complex conjugate of a complex number \( z \) is denoted by \( \overline{z} \) (or might also be written \( z^* \)) and is given by

\[
\overline{z} = x - jy.
\]

Euler’s Identity is

\[
e^{j\theta} = \cos(\theta) + j\sin(\theta).
\]

See its relation to \( \tan^{-1}\left(\frac{y}{x}\right) \) at [https://en.wikipedia.org/wiki/Atan2](https://en.wikipedia.org/wiki/Atan2).

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With this definition, the polar representation of a complex number will make more sense. Note that

$$|z|e^{j\theta} = |z| \cos(\theta) + j|z| \sin(\theta).$$

The reason for these definitions is to exploit the geometric interpretation of complex numbers, as illustrated in Figure 1 in which case $|z|$ is the magnitude and $e^{j\theta}$ is the unit vector that defines the direction.

2 Useful Identities

**Complex Number Properties**

**Rectangular vs. polar forms:** $z = x + jy = |z|e^{j\theta}$

where $|z| = \sqrt{x^2 + y^2}$, $\theta = \text{atan2}(y, x)$. We can also write $x = |z| \cos \theta$, $y = |z| \sin \theta$.

**Euler’s identity:**

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}, \quad \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

**Complex conjugate:** $\overline{z} = x - jy = |z|e^{-j\theta}$

$$z + w = \overline{z} + \overline{w}, \quad (z - w) = \overline{z} - \overline{w}$$

$$zw = \overline{z} \overline{w}, \quad \overline{z/w} = \overline{z}/\overline{w}$$

$z = \overline{z} \iff z$ is real

$$|z^n| = (|z|)^n$$

**Complex Algebra**

Let $z_1 = x_1 + jy_1 = |z_1|e^{j\theta_1}$, $z_2 = x_2 + jy_2 = |z_2|e^{j\theta_2}$.

(Note that we adopt the easier representation between rectangular form and polar form.)

**Addition:** $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

**Multiplication:** $z_1z_2 = |z_1||z_2|e^{j(\theta_1 + \theta_2)}$

**Division:** $\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{j(\theta_1 - \theta_2)}$

**Power:** $z_1^n = |z_1|^n e^{jn\theta_1}$

$$z_1^n = \pm |z_1|^n e^{j\frac{\theta_1}{2}}$$

**Useful Relations**

$$-1 = j^2 = e^{j\pi} = -e^{-j\pi}$$

$$j = e^{j\frac{\pi}{2}} = \sqrt{-1}$$

$$-j = -e^{j\frac{\pi}{2}} = e^{-j\frac{\pi}{2}}$$

$$\sqrt{j} = (e^{j\frac{\pi}{2}})^\frac{1}{2} = \pm e^{j\frac{\pi}{4}} = \pm(1 + j)\sqrt{2}$$

$$\sqrt{-j} = (e^{-j\frac{\pi}{2}})^\frac{1}{2} = \pm e^{-j\frac{\pi}{4}} = \pm(1 - j)\sqrt{2}$$

3 Phasors

We consider sinusoidal voltages and currents of a specific form:

| Voltage   | $v(t) = V_0 \cos(\omega t + \phi_v)$ |
| Current   | $i(t) = I_0 \cos(\omega t + \phi_i)$ |

where,

(a) $V_0$ is the voltage amplitude and is the highest value of voltage $v(t)$ will attain at any time. Similarly, $I_0$ is the current amplitude.

(b) $\omega$ is the angular frequency of oscillation. $\omega$ is related to frequency by $\omega = 2\pi f$. Frequency $f$ is the number of oscillation cycles that occur per second. If $T$ is the period of the sinusoid (that is, the amount of time it takes for one complete cycle to occur) the frequency is $f = \frac{1}{T}$.

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(c) $\phi_v$ and $\phi_i$ are the phase terms of the voltage and current respectively. These capture a delay, or a shift in time, of the sinusoid.

We know from Euler’s identity that $e^{j\theta} = \cos(\theta) + j\sin(\theta)$. Using this identity, we can obtain an expression for $\cos(\theta)$ in terms of an exponential:

$$\cos(\theta) = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$$

Extending this to our voltage signal from above:

$$v(t) = V_0 \cos(\omega t + \phi_v) = \frac{V_0}{2}e^{j\omega t + j\phi_v} + \frac{V_0}{2}e^{-j\omega t - j\phi_v} = \frac{V_0}{2}e^{j\phi_v}e^{j\omega t} + \frac{V_0}{2}e^{-j\phi_v}e^{-j\omega t}$$

The coefficient of the $e^{j\omega t}$ term in Equation 1 is called the phasor form of this signal:

$$\tilde{V} = \frac{1}{2}V_0e^{j\phi_v}$$

The complex conjugate of the phasor is the coefficient of the $e^{-j\omega t}$ term in Equation 1 and is given by:

$$\tilde{V} = \frac{1}{2}V_0e^{-j\phi_v}$$

The phasor representation is a constant that contains the magnitude and phase information of the signal. The time-varying part of the signal does not need to be explicitly represented, because it is given by $e^{j\omega t}$, which is always implicit when using phasors. Phasors let us handle sinusoidal signals much more easily. They are powerful because they let us use DC-like (16A style) circuit analysis techniques, which we already know, to analyze circuits with sinusoidal voltages and currents.

**Note:** We can only use phasors if we know that all of our signals are sinusoids. We will see later in 16B (Module 4) why this is not too restrictive a condition.

Within this standard form, the phasor domain representation is as follows. The general equation that relates cosines to phasors is below, where $\tilde{V}$ is the phasor.

$$V_0 \cos(\omega t + \phi_v) = \tilde{V}e^{j\omega t} + \tilde{V}e^{-j\omega t}$$

The standard forms for voltage and current phasors are given below:

| Voltage   | $\tilde{V} = \frac{1}{2}V_0e^{j\phi_v}$ |
| Current   | $\tilde{I} = \frac{1}{2}I_0e^{j\phi_i}$ |

We define the *impedance* of a circuit component to be $Z = \frac{\tilde{V}}{\tilde{I}}$, where $\tilde{V}$ and $\tilde{I}$ represent the voltage across and the current through the component, respectively. For capacitors and inductors, this impedance will generally depend on $j\omega$. 
3.1 Phasor Relationship for Resistors

Consider a simple resistor circuit as in Figure 2 with current being

\[ i(t) = I_0 \cos(\omega t + \phi) = \frac{I_0}{2} e^{j\phi} e^{j\omega t} + \frac{I_0}{2} e^{-j\phi} e^{-j\omega t} \]

By Ohm’s law,

\[ v(t) = i(t)R = I_0 R \cos(\omega t + \phi) = \frac{I_0 R}{2} e^{j\phi} e^{j\omega t} + \frac{I_0 R}{2} e^{-j\phi} e^{-j\omega t} \]

In the phasor domain, for both \( s = +j\omega \) and \( s = -j\omega \)

\[ \tilde{V} = \tilde{I} \]

We usually refer to the impedance of the resistor, \( Z_R \), in the phasor domain. Since \( Z_R = R \), we can also write:

\[ \tilde{V} = Z_R \tilde{I} \]

3.2 Phasor Relationship for Capacitors

Consider a capacitor circuit as in Figure 3 with voltage being

\[ v(t) = V_0 \cos(\omega t + \phi) \]
By the capacitor equation, and expanding cos in the complex exponential domain:

\[ v(t) = V_0 \cos(\omega t + \phi) \]
\[ = \frac{V_0}{2} e^{j(\omega t + \phi)} + \frac{V_0}{2} e^{-j(\omega t + \phi)} \]
\[ = \frac{V_0}{2} e^{j\phi} e^{j\omega t} + \frac{V_0}{2} e^{-j\phi} e^{-j\omega t} \]
\[ = \tilde{V} e^{j\omega t} + \tilde{V} e^{-j\omega t} \]

\[ i(t) = C \frac{dv}{dt} \]
\[ = C \frac{d}{dt} \left( \frac{V_0}{2} e^{j\omega t} + \frac{V_0}{2} e^{-j\omega t} \right) e^{j\phi} \]
\[ = j\omega C \frac{V_0}{2} e^{j\phi} e^{j\omega t} - j\omega C \frac{V_0}{2} e^{-j\phi} e^{-j\omega t} \]

Considering the \( s = \pm j\omega \) term:

\[ \tilde{I} = j\omega C \frac{V_0}{2} e^{j\phi} = j\omega C \tilde{V} \]

This can also be derived in the sinusoid domain:

\[ i(t) = C \frac{dv}{dt}(t) \]
\[ = -CV_0 \omega \sin(\omega t + \phi) \]
\[ = -CV_0 \omega \left( -\cos\left( \omega t + \phi + \frac{\pi}{2} \right) \right) \]
\[ = CV_0 \omega \cos\left( \omega t + \phi + \frac{\pi}{2} \right) \]
\[ = (\omega C)V_0 \cos\left( \omega t + \phi + \frac{\pi}{2} \right) \]

In the phasor domain,

\[ \tilde{I} = \omega C e^{j\frac{\pi}{2}} \tilde{V} = j\omega C \tilde{V} \]

The impedance of a capacitor is an abstraction to model the capacitor in a manner similar to a resistor in the phasor domain. This is denoted \( Z_C \), which is given by:

\[ Z_C = \frac{\tilde{V}}{\tilde{I}} = \frac{1}{j\omega C} = \frac{1}{sC} \]

for \( s = j\omega \).

1. Cosine vs Sine Relation

   (a) What value of \( \phi \) makes the following relation true?

\[ \sin(\omega t) = \cos(\omega t + \phi) \]
**Answer:** Expanding sin and cos into their complex exponentials:

\[
\sin(\omega t) = \frac{1}{2}e^{j\omega t} - \frac{1}{2}e^{-j\omega t} \\
= -\frac{j}{2}e^{j\omega t} - \frac{j}{2}e^{-j\omega t} \\
= \frac{1}{2}e^{-j\frac{\pi}{2}}e^{j\omega t} + \frac{1}{2}e^{j\frac{\pi}{2}}e^{-j\omega t} \\
= \frac{1}{2}e^{j(\omega t - \frac{\pi}{2})} + \frac{1}{2}e^{-j(\omega t - \frac{\pi}{2})} \\
= \cos\left(\omega t - \frac{\pi}{2}\right)
\]

2. **Inductor Impedance**

Given the voltage-current relationship of an inductor \( V = L \frac{di}{dt} \), show that its complex impedance is \( Z_L = j\omega L \).

**Answer:**

\[
\begin{align*}
\text{Consider a simple resistor circuit as in Figure 4 with current being} & \quad i(t) = I_0 \cos(\omega t + \phi) \\
\text{By the inductor equation,} & \quad v(t) = L \frac{di}{dt}(t) \\
& \quad = -LI_0 \omega \sin(\omega t + \phi) \\
& \quad = LI_0 \omega \cos\left(\omega t + \phi + \frac{\pi}{2}\right) \\
& \quad = (\omega L)I_0 \cos\left(\omega t + \phi + \frac{\pi}{2}\right)
\end{align*}
\]

In the phasor domain,

\[
\tilde{V} = \omega Le^{j\frac{\pi}{2}} \tilde{I} = j\omega L \tilde{I}
\]

The impedance of an inductor is an abstraction to model the inductor as a resistor in the phasor domain. This is denoted \( Z_L \).

\[
Z_L = \frac{\tilde{V}}{\tilde{I}} = j\omega L
\]
3. Complex Matrix Inverse

Consider a complex matrix

\[ M = M_r + jM_i \]

and its inverse

\[ N = N_r + jN_i \]

(a) **Show that the inverse of \( \overline{M} = M_r - jM_i \) (the complex conjugate of \( M \)) is equal to \( \overline{N} = N_r - jN_i \) (the complex conjugate of \( N \)).**

**Answer:** Because \( N \) is the inverse of \( M \), we know

\[ I = NM = (N_r + jN_i)(M_r + jM_i) \]

\[ = (N_r M_r - N_i M_i) + j(N_r M_i + N_i M_r) \]

\[ \Rightarrow I = N_r M_r - N_i M_i \quad 0 = N_r M_i - N_i M_r \]

Note that we isolated the real and imaginary terms in the product. Because the product of a matrix and its inverse must be the identity matrix, the real terms must equal the identity matrix, and the imaginary terms must be 0.

Similarly, we try evaluating the product of \( \overline{N} \overline{M} \):

\[ \overline{N} \overline{M} = (N_r - jN_i)(M_r - jM_i) \]

\[ = (N_r M_r - N_i M_i) - j(N_r M_i + N_i M_r) \]

\[ = I - j0 \]

\[ = I \]

Thus, \( \overline{M}^{-1} = \overline{N} = \overline{M}^{-1} \).

The inverse of the complex conjugate of a matrix is equal to the complex conjugate of the matrix’s inverse.

4. Phasor Analysis

Any sinusoidal time-varying function \( x(t) \), representing a voltage or a current, can be expressed in the form

\[ x(t) = \overline{X} e^{j\omega t} + \overline{X} e^{-j\omega t}, \]

where \( \overline{X} \) is a time-independent function called the **phasor** representation of \( x(t) \) (recall that \( \overline{a} \) denotes the complex conjugate of \( a \)). Note that 1) \( \overline{X} \) and \( \overline{X} \) are complex conjugates of each other, 2) \( e^{j\omega t} \) and \( e^{-j\omega t} \) are complex conjugates of each other, and 3) that \( \overline{X} e^{j\omega t} \) and \( \overline{X} e^{-j\omega t} \) are also complex conjugates of each other.

The phasor analysis method consists of five steps. Consider the RC circuit below.
The voltage source is given by

\[ v_s(t) = 12 \sin \left( \omega t - \frac{\pi}{4} \right), \tag{3} \]

with \( \omega = 1 \times 10^3 \text{ rad/s} \), \( R = \sqrt{3} \text{k}\Omega \), and \( C = 1 \mu\text{F} \).

Our goal is to obtain a solution for \( i(t) \) with the sinusoidal voltage source \( v_s(t) \).

(a) **Step 1: Write sources as exponentials:** \( \bar{A}e^{j\omega t} + \bar{A}e^{-j\omega t} \)

All voltages and currents with known sinusoidal functions should be expressed in the standard exponential format. Convert \( v_s(t) \) into a exponential and write down its phasor representation \( \bar{V}_s \).

**Answer:**

\[
\begin{align*}
v_s(t) &= 12 \sin \left( \omega t - \frac{\pi}{4} \right) \\
&= 12 \left( \frac{1}{2j} e^{j(\omega t - \frac{\pi}{4})} - \frac{1}{2j} e^{-j(\omega t - \frac{\pi}{4})} \right) \\
&= \frac{12}{2} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{4}} e^{j\omega t} + \frac{12}{2} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{4}} e^{-j\omega t} \\
&= 6 e^{-j\frac{\pi}{4}} e^{j\omega t} + 6 e^{j\frac{\pi}{4}} e^{-j\omega t}
\end{align*}
\]

The phasor is given by the term in front of \( e^{j\omega t} \)

\[ \bar{V}_s = 6e^{-j\frac{\pi}{4}} \]

(b) **Step 2: Transform circuits to phasor domain for** \( s = +j\omega \)

The voltage source is represented by its phasor \( \bar{V}_s \). The current \( i(t) \) is related to its phasor counterpart \( \bar{I} \) by

\[ i(t) = \bar{I}e^{j\omega t} + \bar{I}e^{-j\omega t}. \]

What are the impedances of the resistor, \( Z_R \), and capacitor, \( Z_C \)? We sometimes also refer to this as the "phasor representation" of \( R \) and \( C \).

**Answer:**

\[
\begin{align*}
Z_R &= R \\
Z_C &= \frac{1}{j\omega C}
\end{align*}
\]

(c) **Step 3: Cast the branch and element equations in phasor domain**

Use Kirchhoff’s laws to write down a loop equation that relates all phasors in Step 2.

**Answer:**

The KVL equations for this circuit are:

\[ v_s(t) = v_R(t) + v_C(t), \]
where we have denoted the voltage across the resistor as $v_R(t)$. If you expand all these quantities using phasors (using Equation 2), we get:

$$\frac{v_s e^{j\omega t} + v_s e^{-j\omega t}}{v_s(t)} = \frac{V_R e^{j\omega t} + V_R e^{-j\omega t}}{v_R(t)} + \frac{V_C e^{j\omega t} + V_C e^{-j\omega t}}{v_C(t)}$$

Collecting together all the $e^{j\omega t}$ terms and all the $e^{-j\omega t}$ terms, the above equation can be rewritten as

$$(\bar{V}_s - \bar{V}_R - \bar{V}_C)e^{j\omega t} + (\bar{V}_s - \bar{V}_R - \bar{V}_C)e^{-j\omega t} = 0.$$  

If $\omega \neq 0$ it can be shown that both of the coefficients of $e^{j\omega t}$ and $e^{-j\omega t}$ in the above equation must be equal to 0 for this equation to hold. That is:

$$\bar{V}_s - \bar{V}_R - \bar{V}_C = 0, \quad \text{and} \quad \bar{V}_s - \bar{V}_R - \bar{V}_C = 0.$$  

Both of the equations above have the same meaning, i.e., $\bar{V}_s - \bar{V}_R - \bar{V}_C = 0$ or $\bar{V}_s = \bar{V}_R + \bar{V}_C$.

This is exactly the same KVL equation as given by $v_s(t) = v_R(t) + v_C(t)$, but using phasors. In the same way, you can show that KCL is also obeyed by phasors. This conclusion implies that the standard rules for putting together circuit equations using KCL and KVL work identically with phasors as with time-varying notation. The only difference is that the voltage-current relationships of elements should be in phasor form, e.g., $I = j\omega C \bar{V}_C = \frac{1}{Z} \bar{V}_C$.

We can apply what we’ve found above to write the circuit in the phasor domain:

$$Z_R \bar{I} + Z_C \bar{I} = \bar{V}_s$$

$$\left( R + \frac{1}{j\omega C} \right) \bar{I} = 6e^{-\frac{j\pi}{4}}$$

Now that we’ve shown that the phasor representation (i.e., $\bar{I}$ and $\bar{V}$) of our circuit is equivalent to the time-varying representation (i.e., $i(t)$ and $v(t)$), in the future we can write the KCL and KVL equations in phasor form directly.

(d) **Step 4: Solve for unknown variables**

Solve the equation you derived in Step 3 for $\bar{I}$ and $\bar{V}_C$. What is the polar form of $\bar{I}$ and $\bar{V}_C$? Polar form is given by $Ae^{j\theta}$, where $A$ is a positive real number.

**Answer:**

$$\bar{I} = 6e^{-\frac{j\pi}{4}} \frac{R + \frac{1}{j\omega C}}{j\omega RC + 1} = \frac{j6\omega Ce^{-\frac{j\pi}{4}}}{j\omega RC + 1}$$

$$\bar{V}_C = \bar{I}Z_C = \frac{j6\omega Ce^{-\frac{j\pi}{4}}}{1 + j\omega RC} \cdot \frac{1}{j\omega C} = \frac{6e^{-\frac{j\pi}{4}}}{1 + j\omega RC}$$

---

2 Try working out the $\omega = 0$ case by yourself! It’s even easier.

3 This can be shown because the functions $e^{j\omega t}$ and $e^{-j\omega t}$ are linearly independent.
To derive the polar form, we plug in for the values of the circuit elements:

\[ \tilde{I} = \frac{j6\omega Ce^{-j\frac{3\pi}{4}}}{1 + j\omega RC} = \frac{j6 \cdot 10^3 \cdot 10^{-6} \cdot e^{-j\frac{3\pi}{4}}}{1 + j\sqrt{3}} = \frac{(e^{j\frac{\pi}{2}}) \cdot 6 \cdot 10^{-3} \cdot e^{-j\frac{3\pi}{4}}}{1 + j\sqrt{3}} \]

\[ = \frac{6 \cdot 10^{-3} \cdot e^{-j\frac{3\pi}{4}} e^{j\frac{\pi}{2}}}{2e^{j\frac{\pi}{4}}} = 3e^{-j\frac{\pi}{4}} \text{mA}. \]

\[ \tilde{V}_C = \frac{6e^{-j\frac{3\pi}{4}}}{1 + j\omega RC} = \frac{6e^{-j\frac{3\pi}{4}}}{1 + j\sqrt{3}} = \frac{6e^{-j\frac{3\pi}{4}}}{2e^{j\frac{\pi}{4}}} = 3e^{-j\frac{13\pi}{12}} \text{V} \]

(e) **Step 5: Transform solutions back to time domain**

To return to time domain, we apply the fundamental relation between a sinusoidal function and its phasor counterpart. What is \( i(t) \) and \( v_C(t) \)? What is the phase difference between \( i(t) \) and \( v_C(t) \)?

**Answer:**

\[ i(t) = \tilde{I}e^{j\omega t} + \tilde{I}e^{-j\omega t} = 3e^{-j\frac{3\pi}{4}}e^{j\omega t} + 3e^{j\frac{3\pi}{4}}e^{-j\omega t} = 6\cos \left( \omega t - \frac{7\pi}{12} \right) \text{mA} \]

\[ v_C(t) = \tilde{V}_Ce^{j\omega t} + \tilde{V}_Ce^{-j\omega t} = 3e^{-j\frac{13\pi}{12}}e^{j\omega t} + 3e^{j\frac{13\pi}{12}}e^{-j\omega t} = 6\cos \left( \omega t - \frac{13\pi}{12} \right) \text{V} \]

The phase difference between the \( v_C(t) \) and \( i(t) \) with respect to \( i(t) \) is \(-\frac{\pi}{2}\).

5. **RLC Circuit Phasor Analysis**

We study a simple RLC circuit with an AC voltage source given by

\[ v_s(t) = B\cos(\omega t - \phi) \]

(a) Write out the phasor representations of \( v_s(t) \), \( R \), \( C \), and \( L \).

**Answer:**

\[ \tilde{V}_s = \frac{B}{2} e^{-j\phi}, \quad Z_R = R, \quad Z_C = \frac{1}{j\omega C}, \quad Z_L = j\omega L \]

(b) Use Kirchhoff’s laws to write down a loop equation relating the phasors in the previous part.

**Answer:**

\[ Z_R \tilde{I} + Z_C \tilde{I} + Z_L \tilde{I} = \tilde{V}_s \]

\[ \left( R + \frac{1}{j\omega C} + j\omega L \right) \tilde{I} = \frac{B}{2} e^{-j\phi} \]

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(c) Solve the equation in the previous step for the current $\tilde{I}$. What is the magnitude and phase of the polar form of $\tilde{I}$?

**Answer:**

$$
\tilde{I} = \frac{B}{2} e^{-j\phi} \frac{e^{-j\phi}}{R + \frac{1}{j\omega C} + j\omega L} = \frac{B}{2} e^{-j\phi} \frac{e^{-j\phi}}{R + j\left(-\frac{1}{\omega C} + \omega L\right)}
$$

The magnitude of $\tilde{I}$ is

$$
|\tilde{I}| = \frac{B}{2} \sqrt{R^2 + \left(-\frac{1}{\omega C} + \omega L\right)^2}
$$

The phase of $\tilde{I}$ is

$$
\angle \tilde{I} = -\phi - \text{atan2}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)
$$

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