EE16A

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To Do List:

1. Matrix
2. Gaussian Elimination
3. How do you listen in a very loud party?
Matrix: from system of linear equations.

Variables \( x_1, x_2, \ldots, x_n \)

\[
\begin{align*}
\mathbf{a}^T \mathbf{x} &= \mathbf{c} \\
\mathbf{b}^T \mathbf{x} &= \mathbf{c}_2
\end{align*}
\]

Matrix \( n \times n \) vector \( n \times 1 \)
Some special types of matrices

1×1 ? Scalar: height, weight

3×1 ? Vector: location in 3D
            velocity \[ \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \]
            Force

3×3 ! Matrix:

\[ \begin{bmatrix}
  \sigma_{11} & \sigma_{12} & \sigma_{13} \\
  \sigma_{21} & \sigma_{22} & \sigma_{23} \\
  \sigma_{31} & \sigma_{32} & \sigma_{33} 
\end{bmatrix} \]

"Tensor"

\[
\frac{R}{I} = \frac{v}{R}
\]
Special Cases:

0 - matrix

Diagonal

Identity

Upper triangular
How to operate on matrix?

**Scalar:**

**Vector:** $V_1 + V_2$

**Matrix:** $A_1 + A_2$

$\begin{align*}
\text{Scalar:} & \\
\text{Vector:} & V_1 + V_2 \\
\text{Matrix:} & A_1 + A_2
\end{align*}$
**A Simple example (2 variables)**

\[
\begin{align*}
  x_1 + 2x_2 &= 4 \\
  2x_1 - x_2 &= 3
\end{align*}
\]

\[
\begin{align*}
  \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 4 \\ 3 \end{bmatrix} \\
  A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= b
\end{align*}
\]

\[
\begin{align*}
  \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 4 \\ -5 \end{bmatrix} \\
  \begin{bmatrix} x_1 + 2x_2 = 4 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 4 \\ 1 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
  x_2 &= 1 \\
  x_1 &= 2
\end{align*}
\]
more generally → Gaussian Elimination (G.E.)

\[ Ax = b \]

1. linear combinations of rows
2. multiply a row by a scalar
3. swap rows
Scenarios

\[ Ax = b \]

1. \[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  0 & a_{21} & a_{23} \\
  0 & 0 & a_{31}
\end{bmatrix} \rightarrow \text{unique solution}
\]

2. \[
\begin{bmatrix}
  0 & 0 & 0 \\
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} = \begin{bmatrix}
  b_1 \\
  b_2 \\
  b_3
\end{bmatrix} \rightarrow \text{non-unique, \infty many}
\]

   \[ b_3 = 0 \]

3. \[
\begin{bmatrix}
  0 & 0 & 0 \\
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} = \begin{bmatrix}
  b_1 \\
  b_2 \\
  5
\end{bmatrix} \rightarrow \text{inconsistent}
\]
Conversation in a loud party

Friend #1

200

You

X1

\begin{align*}
    a_{11} x_1 + a_{12} x_2 &= b_1 \\
    a_{21} x_1 + a_{22} x_2 &= b_2
\end{align*}

Enemy #2

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