This homework is due February 1, 2019, at 23:59.
Self-grades are due February 5, 2019, at 23:59.

Submission Format
Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

If you do not attach a PDF “printout” of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

1. Administrivia
   If you want to be eligible for midterm clobbering, what do you need to do?

2. Counting Solutions
   For each of the following systems of linear equations, determine if there is a unique solution, no solution, or an infinite number of solutions. Show your work. If there is a unique solution, find it. If there are an infinite number of solutions, describe the space of solutions.

   (a)
   \[
   \begin{align*}
   2x + 3y &= 5 \\
   x + y &= 2 
   \end{align*}
   \]

   (b)
   \[
   \begin{align*}
   x + y + z &= 3 \\
   2x + 2y + 2z &= 5 
   \end{align*}
   \]

   (c)
   \[
   \begin{align*}
   -y + 2z &= 1 \\
   2x + z &= 2 
   \end{align*}
   \]

   (d)
   \[
   \begin{align*}
   x + 2y &= 3 \\
   2x - y &= 1 \\
   3x + y &= 4 
   \end{align*}
   \]

   (e)
   \[
   \begin{align*}
   x + 2y &= 3 \\
   2x - y &= 1 \\
   x - 3y &= -5 
   \end{align*}
   \]
3. Stojanovic’s Optimal Smoothies

Stojanovic’s Optimal Smoothies has a unique way of serving its customers. To ensure the best customer experience, each customer gets a smoothie personalized to their tastes. Professor Stojanovic knows that a lot of customers don’t know what they want, so when the customer walks up to the counter, they are asked to taste four standard smoothies that cover the entire range of flavors found in the smoothies.

Each smoothie is made of \( \frac{1}{2} \) cup Greek yogurt, \( \frac{1}{8} \) cup vanilla soy milk, \( \frac{1}{2} \) cup crushed ice, and 1 cup mystery fruit. The four standard smoothies have the following recipes for the cup of mystery fruit:

<table>
<thead>
<tr>
<th>Fruit [cups]</th>
<th>Banana Berry</th>
<th>Caribbean Passion</th>
<th>Mango-a-go-go</th>
<th>Strawberries Wild</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberries</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>Bananas</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>Mangos</td>
<td>0</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{5}{3} )</td>
<td>0</td>
</tr>
<tr>
<td>Blueberries</td>
<td>( \frac{1}{3} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Each customer is assumed to have a score (from 0 to 10) for each fruit, and the total score for the smoothie is computed by multiplying the score for a fruit with its proportion in the smoothie. The other ingredients do not influence the customer’s score. For example, if a customer’s score for strawberries is 6 and bananas is 3, then the total score for the Strawberries Wild smoothie would be \( 6 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} = 5 \).

After a customer gives a score (from 0 to 10) for each smoothie, Professor Stojanovic then calculates (on the spot!) how much the customer likes each fruit. Then Professor Stojanovic blends up a special smoothie that will maximize the customer’s score.

Professor Liu was thirsty after giving the first lecture, so Professor Liu decided to take a drink break at Stojanovic’s Optimal Smoothies. Professor Liu walked in and gave the following ratings:

<table>
<thead>
<tr>
<th>Smoothie</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banana Berry</td>
<td>7</td>
</tr>
<tr>
<td>Caribbean Passion</td>
<td>7</td>
</tr>
<tr>
<td>Mango-a-go-go</td>
<td>7( \frac{2}{3} )</td>
</tr>
<tr>
<td>Strawberries Wild</td>
<td>6( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

(a) What were Professor Liu’s ratings for each fruit? **Work this problem out by hand in terms of the steps. You may use a calculator to do algebra.**

(b) What mystery fruit combination should Professor Stojanovic put in Professor Liu’s personalized smoothie to maximize the customer’s score? What score would Professor Liu give for this smoothie? There may be more than one correct answer.

4. Filtering Out The Troll

You attended a very important public speech and recorded it using a recording device that consists of two directional microphones. However, there was this particular person in the audience who was trolling around. When you went back home to listen to the recording, you realized that the two recordings were dominated by the troll and you could not hear the speech. Fortunately, since you had two microphones, you realized that there is a way to combine the two recordings such that the trolling is removed. Recollecting the scene, the locations of the speaker and the troll are shown in Figure 1.
The way your recording device works is that each microphone weighs the audio signal depending on the angle of the audio source, relative to the x axis, hence the name *directional microphones*. More specifically, if the audio source is located at an angle of $\theta$, the first microphone will record the audio signal with weight $f_1(\theta) = \cos(\theta)$, and the second microphone will record the audio signal with weight $f_2(\theta) = \sin(\theta)$. For example, an audio source that lies on the $x$ axis will be recorded with the first microphone with weight equal to 1 (since $\cos(0) = 1$), but will not be picked up by the second microphone (since $\sin(0) = 0$). Note that the weights can also be negative.

Graphically, the directional characteristics of the microphones are given in Figures 2 and 3 (the red and blue colors denote the positive and negative values of the weight, respectively and the distance of the red or the blue line from the midpoint is the value of the weight). Putting all of this together, assume that there are two speakers, $A$ and $B$, at angles $\theta$ and $\psi$, respectively. Assume that speaker $A$ produces an audio signal represented by the vector $\vec{a} \in \mathbb{R}^n$. That is, the $i$-th component of $\vec{a}$ is the signal at the $i$-th time step. Similarly, assume speaker $B$ produces an audio signal $\vec{b}$.

Then the first microphone will record the signal

$$\vec{m}_1 = \cos(\theta) \cdot \vec{a} + \cos(\psi) \cdot \vec{b},$$

and the second microphone will record the signal

$$\vec{m}_2 = \sin(\theta) \cdot \vec{a} + \sin(\psi) \cdot \vec{b}. $$
Figure 2: Directional characteristics of mic. 1. The red and blue colors denote the positive and negative values of the weight, respectively. The distance of the red or the blue line from the midpoint is the value of the weight.

Figure 3: Directional characteristics of mic. 2. The red and blue colors denote the positive and negative values of the weight, respectively. The distance of the red or the blue line from the midpoint is the value of the weight.

(a) Using the notation above, let the important speaker be speaker $A$ (with signal $\vec{a}$) and let the person trolling be speaker $B$ (with signal $\vec{b}$). Express the recordings of the two microphones $\vec{m}_1$ and $\vec{m}_2$ (i.e. the signals recorded by the first and the second microphones, respectively) as a linear combination of \( \vec{a} \) and $\vec{b}$.

(b) Recover the important speech $\vec{a}$, as a weighted combination of $\vec{m}_1$ and $\vec{m}_2$. In other words, write $\vec{a} = u \cdot \vec{m}_1 + v \cdot \vec{m}_2$ (where $u$ and $v$ are scalars). What are the values of $u$ and $v$?

(c) Partial IPython code can be found in `prob1.ipynb`. Complete the code to get a clean signal of the important speech. What does the speaker say? (Optional: Where is the speech taken from?)

Note: You may have noticed that the recordings of the two microphones sounded remarkably similar. This means that you could recover the real speech from two “trolled” recordings that sounded almost identical! Leave out the fact that the recordings are actually different, and have some fun with your friends who aren’t lucky enough to be taking EE16A.

5. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID’s. (In case of homework party, you can also just describe the group.) How did you work on this homework?