Golden Rules Revisited

Recall for an ideal op amp, the "golden rules" are

• (1) \( i_+ = i_- = 0 \). No current can flow into the positive or negative input terminal. **Note that this rule holds regardless of whether there is negative feedback or not.**

• (2) For a circuit with negative feedback, \( v_+ = v_- \). One important thing to note now is this rule only holds **when there is negative feedback**. 
  \( v_+ = v_- \) means that the input voltage \( v_{in} = v_+ - v_- \) is 0V for an op-amp. In order to understand \( V_{out} \) can be non-zero for \( v_{in} = 0 \), let’s review the block diagram of an op amp in negative feedback.

\[
\begin{align*}
\text{We have negative feedback gain } f. \text{ For an op-amp:} \\
V_c &= \frac{1}{1 + A \cdot f} \times v_{in}. \\
\text{For an op-amp in negative feedback, we have} \\
v_{in} - f \cdot V_{out} &= V_c. \\
\text{We know } V_{out} = A \cdot V_c \text{ and we can derive:} \\
V_{out} &= \frac{A}{1 + A \cdot f} V_c. \\
\text{When } v_{in} = 0V, V_c = 0V. \text{ Recall mathematically, } \lim_{x \to \infty} k \cdot x \cdot \frac{1}{x} = k \text{ even though } \lim_{x \to \infty} \frac{1}{x} = 0. \\
\text{Similarly, for an ideal op-amp, } V_{out} \text{ can be non-zero even if } V_c = 0 \text{ given } A = \infty.
\end{align*}
\]
Inverting Amplifier & Negative Feedback

Now let's use the golden rules to analyze the circuit we saw earlier:

Let's assume that this circuit is in negative feedback so that we can use golden rule #2, after analyzing the circuit we will show how to check if a circuit is indeed in negative feedback.

We know that by applying KCL, we have

\[ i_1 = i_2 + i_- \]  \hspace{1cm} (4)

\[ i_{\text{out}} = i_2. \]  \hspace{1cm} (5)

Using golden rules # 1, we have

\[ i_- = 0. \]  \hspace{1cm} (6)

Thus, \( i_1 = i_2 \). Using golden rules # 2, we have

\[ v_- = v_+ = 0. \]  \hspace{1cm} (7)

The second equation indicates \( u_2 = 0 \).

Using Ohm's law, we have

\[ u_1 - u_2 = i_1 R_1. \]  \hspace{1cm} (8)

\[ u_2 - u_3 = i_2 R_2. \]  \hspace{1cm} (9)

After solving the equations, we get

\[ V_{\text{out}} = -\frac{R_2}{R_1} v_{\text{in}}. \]  \hspace{1cm} (10)

This circuit is known as "inverting amplifier". It is called "inverting" because it amplifies \( v_{\text{in}} \) by some negative factor \( -\frac{R_2}{R_1} \).

We have used golden rule #2 to simplify circuit analysis, now we will show how to check if a circuit is in negative feedback:
• **Step 1** Zero out all independent sources. For any independent sources that outputs a constant voltage or current, we will zero them out like we did in Thevenin-Norton Equivalencies.

• **Step 2** "Dink" the output, check if the feedback coming from the circuit as a result of "dinking" the output has the opposite direction of "dinking". If the feedback coming from the circuit is in the opposite direction as "dinking" the output, then the circuit is in negative feedback. If not, the circuit is in positive feedback or zero feedback.

Let's use the rules to check some example circuits.

**Example circuit 1**

![Circuit Diagram](image)

**Step 1: zero-out all independent sources** We can zero-out the independent source $v_{in}$ in the circuit, and the circuit diagram becomes:

![Circuit Diagram](image)

**Step 2** "dink $V_{out}$" If we increase $V_{out}$, we know $u_1$ is going to increase because $R_1$ and $R_2$ form a voltage divider circuit. $u_1$ is connected to $v_-$, so $v_-$ increases, therefore $v_{in} = v_+ - v_-$ decreases, and thus $V_{out}$ decreases. So the feedback from the circuit is in the opposite direction, this circuit is in negative feedback.

**Example circuit 2**
Step 1 Zero out all independent sources.

Step 2 "dink $V_{out}$" If we increase $V_{out}$, we know $u_1$ is going to increase because $u_1 = V_{out}$. $u_1$ is connected to $v_+$, so $v_+$ increases, therefore $v_{in} = v_+ - v_-$ increases, and thus $V_{out}$ increases. So the feedback from the circuit is in the same direction, this circuit is in positive feedback. Example circuit 3

Step 1 Zero out all independent sources. We cannot zero out controlled voltage source $-1 \cdot V_f$ because this is not an independent voltage source.
Step 2 "dink V_{out}" If we increase V_{out}, V_f increases which then decreases v_- = -1 \cdot V_f, which then increases v_{in} = v_+ - v_-, thus V_{out} increases. The circuit response to the increase in V_{out} is in the same direction as we "dinked" V_{out}. So this circuit is not in negative feedback.

Non-inverting & Inverting Amplifiers

There are two important categories of amplifier circuits that you should be familiar with, they are non-inverting amplifier and inverting amplifier:

- Non-inverting Amplifier

For the non-inverting amplifier circuit, we have

\[ V_{out} = \left(1 + \frac{R_2}{R_1}\right) \cdot v_{in}. \]  

(11)

- Inverting Amplifier
For the inverting amplifier circuit, we have

\[ V_{\text{out}} = -\frac{R_2}{R_1} \cdot v_{\text{in}}. \]  

(12)

**Artificial Neuron**

The underlying mathematical operations for artificial intelligence is matrix multiplication, which in essence is vector multiplication:

\[
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
\cdot
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
= a_1 \cdot x_1 + a_2 \cdot x_2
\]  

(13)

This vector multiplication can be realized by the following circuit model:

First, let’s apply the first golden rule, \( i_- = i_+ = 0 \). Applying KCL at the node labelled \( v_- \), we have

\[ i_1 + i_2 = i_- + i_3 = 0 + i_3 = i_3. \]  

(14)
Hence, we have
\[ i_1 + i_2 = i_3. \]  
(15)

Now by the second golden rule and the fact that the positive input terminal is connected to ground, we have
\[ v_+ = v_- = 0. \]  
(16)

Applying Ohm’s law at each of the three resistors, we have
\[ i_1 = \frac{V_1 - v_-}{R_1} = \frac{V_1}{R_1} \]  
(17)
\[ i_2 = \frac{V_2 - v_-}{R_2} = \frac{V_2}{R_2} \]  
(18)
\[ i_3 = \frac{v_- - V_{out}}{R_3} = -\frac{V_{out}}{R_3} \]  
(19)
(20)

Plugging in the above result to the KCL equation \( i_1 + i_2 = i_3 \) derived previously, we have
\[ \frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_{out}}{R_3}. \]  
(21)

Multiplying both sides by \( R_3 \), we have
\[ V_{out} = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2, \]  
(22)
relating the final output with the two inputs \( V_1 \) and \( V_2 \), which are \( x_1 \) and \( x_2 \) in equation (13). Now, \( a_1 = \frac{R_3}{R_1} \), \( a_2 = \frac{R_3}{R_2} \). This circuit is named "inverting summing amplifier". Because \( R_1, R_2, R_3 \) are resistors and all have positive values, the coefficients \( a_1 \) and \( a_2 \) are all negative. How do we make \( a_1 \) and \( a_2 \) positive instead? By adding another inverter amplifier circuit at \( V_{out} \), we can get positive coefficients.
Now we have

\[ V_{ana} = \frac{-R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2. \] (23)

For the inverting amplifier just added

\[ V_{out} = -\frac{R_4}{R_4} V_{ana}. \] (24)

Hence,

\[ V_{out} = \frac{R_3}{R_1} V_1 + \frac{R_3}{R_2} V_2. \] (25)

Now we have positive coefficients \( a_1 = \frac{R_3}{R_1}, a_2 = \frac{R_3}{R_2}. \)