**This homework is due October 9, 2017, at 23:59.**
**Self-grades are due October 12, 2017, at 23:59.**

**Submission Format**
Your homework submission should consist of one file.

- **hw6.pdf**: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Circuit Analysis**

Using the steps outlined in lecture, solve the following circuits. You may use a numerical tool, such as IPython.

(a) $V_s = 5 \text{ V}, I_s = 2 \text{ A}, R_1 = R_2 = 2 \Omega, R_3 = 4 \Omega$

![Circuit Diagram](image)

**Solution:**
First, select a ground node.

![Circuit Diagram](image)

Then label all of the node potentials.
Now we label all of the branch currents.

Now, we label all of the voltages across the elements according to passive sign convention.

Now, we have 8 unknowns in this system: $u_1, u_2, u_3, i_{V_s}, i_{I_s}, i_{R_1}, i_{R_2}, i_{R_3}$. Therefore, our vector of unknowns is of size 8, and the matrix will be $8 \times 8$.

Using KCL, we get the following equations:

$$i_{V_s} + i_{R_1} = 0$$
$$-i_{R_1} + i_{R_2} + i_{R_3} = 0$$
$$-i_{R_2} + i_s = 0$$

From the IV relations for all of the elements, we find the following equations:

$$u_1 - 0 = V_s$$
$$i_s = I_s$$
$$u_1 - u_2 - R_1 i_{R_1} = 0$$
$$u_2 - u_3 - R_2 i_{R_2} = 0$$
$$u_2 - 0 - R_3 i_{R_3} = 0$$
Note that we now have 8 equation for 8 unknowns. Thus, we set up the following matrix relation:

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -R_1 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & -R_2 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & -R_3 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
i_{v_s} \\
i_{r_1} \\
i_{r_2} \\
i_{r_3} \\
i_t \\
u_1 \\
u_2 \\
u_3 \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Finally, we plug in the values we were given into the matrix above and use Gaussian elimination to find the vector of unknowns.

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & -4 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
i_{v_s} \\
i_{r_1} \\
i_{r_2} \\
i_{r_3} \\
i_t \\
u_1 \\
u_2 \\
u_3 \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
5 \\
2 \\
0 \\
0 \\
\end{bmatrix}
\]

We find that:

\[
\begin{bmatrix}
i_{v_s} \\
i_{r_1} \\
i_{r_2} \\
i_{r_3} \\
i_t \\
u_1 \\
u_2 \\
u_3 \\
\end{bmatrix} =
\begin{bmatrix}
-2.167 \\
2.167 \\
2 \\
0.167 \\
2 \\
5 \\
0.667 \\
-3.33 \\
\end{bmatrix}
\]

(b) \(V_s = 5\text{ V}, R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 3\Omega, R_4 = 4\Omega, R_5 = 5\Omega\)

Solution:
Here, we will skip showing all of the individual steps. Below is the circuit with our choice of ground and current directions.
From the above circuit, we get the following KCL equations:
\[ i_V + i_{R_1} + i_{R_3} = 0 \]
\[ -i_{R_1} + i_{R_2} + i_{R_5} = 0 \]
\[ -i_{R_3} + i_{R_4} - i_{R_5} = 0 \]

Using the IV relations for each element, we find 6 more equations:
\[ u_1 - 0 = V_s \]
\[ u_1 - u_2 - i_{R_2}R_1 = 0 \]
\[ u_2 - 0 - i_{R_2}R_2 = 0 \]
\[ u_1 - u_3 - i_{R_2}R_3 = 0 \]
\[ u_3 - 0 - i_{R_4}R_4 = 0 \]
\[ u_2 - u_3 - i_{R_5}R_5 = 0 \]

Note that we now have 9 equation for 9 unknowns. Thus, we set up the following matrix relation:

\[
\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -R_1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & -R_2 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -R_3 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & -R_4 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -R_5 & 0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
i_V \\
i_{R_1} \\
i_{R_2} \\
i_{R_3} \\
i_{R_4} \\
i_{R_5} \\
i_1 \\
i_2 \\
i_3
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
V_s \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Finally, we plug in the values we were given into the matrix above and use Gaussian elimination to
find the vector of unknowns.

\[
\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -5 & 0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
i_{V_s} \\
i_{R_1} \\
i_{R_2} \\
i_{R_3} \\
i_{R_4} \\
i_{R_5} \\
u_1 \\
u_2 \\
u_3
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
5 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

2. Midterm Problem 3
Redo Midterm Problem 3.

Solution: See midterm solutions.

3. Midterm Problem 4
Redo Midterm Problem 4.

Solution: See midterm solutions.

4. Midterm Problem 5
Redo Midterm Problem 5.

Solution: See midterm solutions.

5. Midterm Problem 6
Redo Midterm Problem 6.

Solution: See midterm solutions.

6. Midterm Problem 7
Redo Midterm Problem 7.

Solution: See midterm solutions.

7. Midterm Problem 8
Redo Midterm Problem 8.

Solution: See midterm solutions.

8. Midterm Problem 9
Redo Midterm Problem 9.

Solution: See midterm solutions.
9. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous “Bloom’s Taxonomy” that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don’t want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don’t have to achieve this every week. But unless you try every week, it probably won’t ever happen.

10. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID’s. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.