- Electron Energy Band
- Fermi Level
- Electrostatics of device charges
The Simplified Electron Energy Band Diagram

- **Energy band diagram** shows the bottom edge of conduction band, $E_c$, and top edge of valence band, $E_v$.
- $E_c$ and $E_v$ are separated by the *band gap energy*, $E_g$. 
Energy Band Diagram with $E$-field

Electron concentration $n$

$$\frac{n(2)}{n(1)} = \frac{e^{q\phi(2)/kT}}{e^{q\phi(1)/kT}} = e^{q[\phi(2)-\phi(1)]/kT}$$
The Fermi-Dirac Distribution (Fermi Function)

Probability of available states at energy $E$ being occupied

$$f(E) = \frac{1}{1 + \exp \left( \frac{E - E_f}{kT} \right)}$$

where $E_f$ is the Fermi energy and $k = \text{Boltzmann constant}=8.617 \times 10^{-5} \text{ eV/K}$
Properties of the Fermi-Dirac Distribution

Probability of electron state at energy E will be occupied

(1) \( f(E) \approx \exp \left[- \frac{(E - E_F)}{kT} \right] \) for \((E - E_F) > 3kT\)

• This approximation is called Boltzmann approximation

(2) Probability of available states at energy E NOT being occupied

\[ 1 - f(E) = \frac{1}{1 + \exp \left( \frac{E_F - E}{kT} \right)} \]

Note:
At 300K, \( kT = 0.026\text{eV} \)
How to find $E_f$ when $n$ (or $p$) is known

\[ \frac{E_c}{E_f} = \frac{E_i}{q|\Phi_F|} \quad \text{(n-type)} \]

\[ \frac{E_f}{E_v} \quad \text{(p-type)} \]

\[ n = n_i \exp \left[ \frac{(E_f - E_i)}{kT} \right] \]

Let $q\Phi_F = E_f - E_i$

\[ \therefore n = n_i \exp \left[ \frac{q\Phi_F}{kT} \right] \]
Dependence of Fermi Level with Doping Concentration

\[ E_i \equiv \frac{(E_C + E_V)}{2} \text{  Middle of energy gap} \]

When Si is undoped, \( E_f = E_i \); also \( n = p = n_i \)
The Fermi Energy at thermal equilibrium

At thermal equilibrium (i.e., no external perturbation),
The Fermi Energy must be constant for all positions

Electron energy

Material A  Material B  Material C  Material D

$E_F$

Position x
Electron Transfer during contact formation

Before contact formation

System 1: $E_{F1}$
System 2: $E_{F2}$

After contact formation

System 1: $E_F$  Net positive charge +
System 2: Net negative charge -

Net negative charge
Net positive charge
Fermi level of the side which has a relatively higher electric potential will have a relatively lower electron energy (Potential Energy = \(-q \cdot \text{electric potential}\).) Only difference of the E's at both sides are important, not the absolute position of the Fermi levels.

Potential difference across depletion region

\[= V_{bi} - V_a\]
PN junctions

Thermal Equilibrium

Complete Depletion Approximation used for charges inside depletion region

\[ r(x) \approx N_D^+(x) - N_A^-(x) \]

http://jas.eng.buffalo.edu/education/pn/pnformation2/pnformation2.html
Electrostatics of Device Charges

1) Summation of all charges = 0

\[ \rho_2 \cdot x_{d2} = \rho_1 \cdot x_{d1} \]

2) \( E \)-field = 0 outside depletion regions
3) Relationship between $E$-field and charge density $\rho(x)$

$$d \left[ \varepsilon E(x) \right] /dx = \rho(x) \quad \text{“Gauss Law”}$$

4) Relationship between $E$-field and potential $\phi$

$$E(x) = - d\phi(x)/dx$$
Example Analysis: n+/ p-Si junction

1) \( Q' = qN_a x_d \)

2) \( E = 0 \)

3) Slope = \( qN_a / \varepsilon_s \)

4) Area under E-field curve = voltage across depletion region = \( qN_a x_d^2 / 2\varepsilon_s \)

Depletion region is very thin and is approximated as a thin sheet charge
Superposition Principle

If \( \rho_1(x) \Rightarrow E_1(x) \) and \( V_1(x) \)
\( \rho_2(x) \Rightarrow E_2(x) \) and \( V_2(x) \)
then
\( \rho_1(x) + \rho_2(x) \Rightarrow E_1(x) + E_2(x) \) and \( V_1(x) + V_2(x) \)

\[ \rho(x) \]

\[ -x_p \]

\[ -qN_A \quad +qN_D \]

\[ x=0 \]

\[ x \]

\[ Q=+qN_A x_p \]

\[ x \]

\[ Q=-qN_A x_p \]

\[ x=0 \]
\[ E_1(x) \]
\[ \rho_1(x) \]
\[ x=0 \]
\[ Q=+qN_A x_p \]
\[ -qN_A \]
\[ -x_p \]
\[ \text{Slope} = -\frac{qN_A}{\varepsilon_s} \]

\[ E_2(x) \]
\[ \rho_2(x) \]
\[ x=0 \]
\[ Q=-qN_A x_p \]
\[ +x_n \]
\[ +qN_D \]
\[ \text{Slope} = +\frac{qN_D}{\varepsilon_s} \]
\[ E(x) = E_1(x) + E_2(x) \]

Sketch of \( E(x) \)

Slope = \(-\frac{qN_A}{\varepsilon_s}\)

Slope = \(+\frac{qN_D}{\varepsilon_s}\)

\[ E_{\text{max}} = -\frac{qN_A x_p}{\varepsilon_s} = -\frac{qN_D x_n}{\varepsilon_s} \]
Depletion Mode: Charge and Electric Field Distributions by Superposition Principle of Electrostatics
Why $x_{d_{\text{max}}}$ ~ constant beyond onset of strong inversion?

Approximation assumes $V_{Si}$ does not change much

$V_G = V_{FB} + V_{OX} + V_{Si}$

Higher than $V_T$
Picks up all the changes in $V_G$

Approximation assumes $V_{Si}$ does not change much

Justification:
If surface electron density changes by $\Delta n$

$\Delta V_{OX} \propto \frac{\Delta n}{C_{OX}}$

but the change of $V_{Si}$ changes only by $kT/q \ln (\Delta n)$ – small!
N (surface) = n (bulk) \cdot \exp [ qV_{Si}/kT]

p-Si

N_a = 10^{16}/cm^3

n-bulk = 2.1 \cdot 10^4/cm^3

V_{Si} | n-surface
---|---
0 | 2.10E+04
0.1 | 9.84E+05
0.2 | 4.61E+07
0.3 | 2.16E+09
0.4 | 1.01E+11
0.5 | 4.73E+12
0.6 | 2.21E+14
0.7 | 1.04E+16
0.8 | 4.85E+17
0.9 | 2.27E+19

Onset of strong inversion (at V_T)