

Homework Assignment # 11 Solutions

Note :

$|\phi_F|$ is sometimes labeled as $|f_p|$ or $|f_n|$ in equations (representing the value of $|f_F|$ for p-type and n-type substrates respectively)

Problem 1

(a) NMOS

$$|\phi_F| = 0.026 \ln \frac{2 \times 10^{16}}{1.45 \times 10^{10}} = 0.37 \text{ V}$$

$$V_{FB} = \phi_{MS} = 4.15 - (4.15 + 0.56 + 0.37) = -0.93 \text{ V}$$

$$V_{Si} \text{ at onset of strong inversion} = 2 |\phi_F| = +0.74 \text{ V}$$

$$x_{dmax} = \left(\frac{2\epsilon_s \cdot 2 |\phi_F|}{qN_a} \right)^{1/2} = 0.22 \text{ } \mu\text{m}$$

$$Q_{dmax} = N_a \cdot x_{dmax} = 4.4 \times 10^{11} \text{ q/cm}^2$$

$$x_{ox} = 4.5 \times 10^{-6} \text{ cm implies } C_{ox} = 7.7 \times 10^{-8} \text{ F/cm}^2$$

$$V_{ox} = \frac{Q_{dmax}}{C_{ox}} = +0.91 \text{ V}$$

$$\therefore V_{TN} = V_{FB} + V_{si} + V_{ox} = -0.93 + 0.74 + 0.91 = +0.72 \text{ V}$$

(b) PMOS

$$V_{FB} = 4.15 - (4.15 + 0.56 - 0.37) = -0.19 \text{ V}$$

$$V_{Si} \text{ at onset of strong inversion} = -2 |\phi_F| = -0.74 \text{ V (minus because of opposite charge of depletion region)}$$

$$V_{ox} = - \frac{Q_{dmax}}{C_{ox}} = -0.91 \text{ V (minus because of opposite charge of depletion region)}$$

$$\therefore V_{TP} = V_{FB} + V_{si} + V_{ox} = -0.19 - 0.74 - 0.91 = -1.84 \text{ V}$$

Problem 2

$$a.) |\phi_p| = 0.026 \ln \frac{5 \times 10^{15}}{1.45 \times 10^{10}} = 0.332 \text{ volts } \therefore \phi_{MS} = -0.56 - |\phi_p| = -0.892 \text{ volts}$$

$$\therefore V_T = \phi_{MS} - \frac{Q_f}{C_{ox}} + 2 |\phi_p| + \frac{\sqrt{4\epsilon_s q N_a |\phi_p|}}{C_{ox}}$$

$$\Rightarrow 1.0 = -0.892 + 0.664 + \frac{1}{C_{ox}} [-1.6 \times 10^{-19} \times 3 \times 10^{10} + (4 \times 1.04 \times 10^{-12} \times 1.6 \times 10^{-19} \times 5 \times 10^{15} \times 0.332)^{1/2}]$$

$$\Rightarrow C_{ox} = 2.33 \times 10^{-8} \text{ F/cm}^2 \quad \therefore x_{ox} = \frac{\epsilon_{ox}}{C_{ox}} = \frac{3.45 \times 10^{-13} \text{ F/cm}^2}{2.33 \times 10^{-8} \text{ F/cm}^2} = 1480 \text{ } \text{\AA} = 148 \text{ nm}$$

b.) Phosphorus (donor dopant) implant will give a negative V_T shift.

$$\Delta V_T = 1 - (-2) = 3 \text{ volts (shifted from +1 volt to -2 volts)}$$

$\therefore \Delta V_T = \frac{q\phi}{C_{ox}}$ by assuming all implanted phosphorus in Si are inside Si and localized at Si-SiO₂ interface.

$$\therefore \text{Implant dose } f = \frac{\Delta V_T \cdot C_{ox}}{q} = \frac{3 \times 2.33 \times 10^{-8}}{1.6 \times 10^{-19}} = 4.4 \cdot 10^{11} / \text{cm}^2$$

Problem 3

$$C_{ox} = 1.57 \times 10^{-7} \text{ F/cm}^2$$

With $N_a = 2 \times 10^{16}/\text{cm}^3$ and $N_d = 10^{16}/\text{cm}^3$

$$\phi_n = 0.35 \text{ V}$$

$$\phi_p = 0.37 \text{ V}$$

$$V_{TN} = (4.15 - 4.15 - 0.56 - 0.37) + 2 \times 0.37 + 0.45 = +0.26 \text{ V}$$

$$V_{TP} = (4.15 - 4.15 - 0.56 + 0.35) - 2 \times 0.35 - 0.3 = -1.21 \text{ V}$$

Goal is to make $V_{TN}(\text{new}) = -V_{TP}(\text{new})$ after threshold implant

The same implant will give identical V_T shift for both NMOS and PMOS.

$$\text{Therefore } V_{TN} + \Delta V_T = - (V_{TP} + \Delta V_T)$$

We will find out later what specie is required from the algebraic sign of ΔV_T .

$$\Delta V_T = (1.21 - 0.26) / 2 = +0.475 \text{ V}$$

(+ shift means p-dopant implant such as Boron)

$$\text{Boron dose} = \frac{0.475 \times 1.57 \times 10^{-7}}{1.6 \times 10^{-19}} = 4.66 \times 10^{11} / \text{cm}^2$$

Problem 4

$$I_D = \mu_n C_{ox} W/L (V_G - V_T) V_{DS} \text{ for small } V_{DS}$$

(a) From the intercepts on the x-axis, $V_T(V_B = 0) = 1.5 \text{ V}$, $V_T(V_B = -2 \text{ V}) = 3.5 \text{ V}$

(b) $\Delta V_T = \gamma [(2|\phi_p| + V_{SB})^{1/2} - (2|\phi_p|)^{1/2}]$ where the body coefficient $\gamma = \frac{\sqrt{2\epsilon_s q N_a}}{C_{ox}}$

Since $|\phi_p| = 0.026 \ln \left(\frac{N_a}{1.45 \times 10^{10}} \right)$ and with $V_{SB} = 2 \text{ V}$, we can solve for N_a by iteration.

$$N_a \sim 2 \times 10^{16} / \text{cm}^3$$

(c) The slope of the I_D versus V_G curves is $\mu_n C_{ox} (W/L) V_{DS} = 10 \mu\text{A/V}$

With $C_{ox} = 3.45 \times 10^{-8} \text{ F/cm}^2$, $W/L = 10$, $V_{DS} = 50 \text{ mV}$

$$\mu_n \sim 580 \text{ cm}^2/\text{V-sec}$$

Note: For carrier mobility of a MOSFET channel, one cannot simply use the mobility curves due to dopant concentration. Scattering mechanisms at the oxide-Si interface usually reduces the mobility. The rule of thumb is approximating the channel mobility as 1/2 of the bulk mobility.

(d) We know $V_T(V_B = 0) = 1.5 \text{ V}$

For $V_G = 10 \text{ V}$, $I_{Dsat} = \frac{\mu_n C_{ox} W}{2L} (V_G - V_T)^2 \sim 7.2 \text{ mA}$.

Problem 5

(a) $\frac{C_{min}}{C_{ox}} = 0.52$ from graph = $\frac{1}{1 + \frac{C_{ox} x_{dmax}}{\epsilon_s}} \therefore \frac{C_{ox} x_{dmax}}{\epsilon_s} = 0.923$

$$x_{dmax} = 0.72 \text{ nm}$$

$$(b) \frac{q N_a x_{dmax}^2}{2\epsilon_s} = 2|\phi_p| = 2 \frac{kT}{q} \ln \frac{N_a}{n_i}$$

$$\therefore \frac{N_a}{\ln \frac{N_a}{1.45 \times 10^{10}}} = 1.3 \times 10^{14}$$

By iteration, $N_a \sim 1.5 \times 10^{15} / \text{cm}^3$

(c) $V_T = -3V$, $C_{ox} = 1.33 \times 10^{-8} \text{ F/cm}^2$, $|\phi_p| = 0.3V$

$$\text{From } V_T = \phi_{MS} - \frac{Q_f}{C_{ox}} + 2|\phi_p| + \frac{\sqrt{4\epsilon_s q N_a |\phi_p|}}{C_{ox}}, \phi_{MS} = 0.57V$$

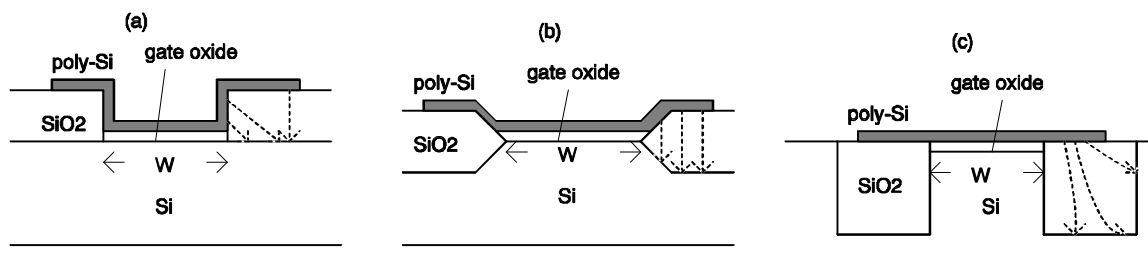
Since $\phi_s = 4.91 \text{ V}$

$$\therefore qf_M = 4.34eV$$

Problem 6

The narrow width effect is caused by the excess depletion charge in Si which the gate has to create outside the W region (V_T will go up as compared with the ideal case). Since the LOCOS structure (b) has the most gradual rise of oxide thickness around the perimeter, the gate will have the most coupling with the substrate Si to create excess depletion charges. The oxide cut (a) has an oxide slope of 90° , the effective coupling is less. For the oxide trench (c), there is no extra Si substrate next to the channel to create excess depletion charge. *Structure (c) will exhibit the least narrow width effect.*

The gate coupling to the Si substrate can also be visualized by the length of the dashed E-field lines (the longer the lines, the less coupling). In terms of narrow width effect, ranking in decreasing order will be : LOCOS, oxide cut, oxide trench.



Problem 7

(a) $V_G \text{ max} = 8 \times 10^6 \text{ V/cm} \times 2 \times 10^{-6} \text{ cm} = 16 \text{ V}$

(b) $C_{ox} = 1.725 \times 10^{-7} \text{ F/cm}^2$

With $N_a = 10^{16} / \text{cm}^3$

$$|\phi_F| = 0.35\text{V}$$

$$V_{TN} = -0.56 + |\phi_F| + \frac{\sqrt{4\epsilon_s q N_a |\phi_F|}}{C_{ox}} = +0.07\text{V}$$

$$(c) \ x_{dmax} = \sqrt{\frac{4\epsilon_s |\phi_F|}{q N_a}} = 3 \cdot 10^{-5}\text{cm}$$

(d) $V_G - V_T = 5 - 0.08 < V_D = 5\text{V}$. *Transistor is in saturation mode.*

$$I_{Dsat} = k[(V_G - V_T)^2/2] = 605\text{mA}$$

(e) A boron threshold implant will shift the V_T for more positive threshold voltage values.

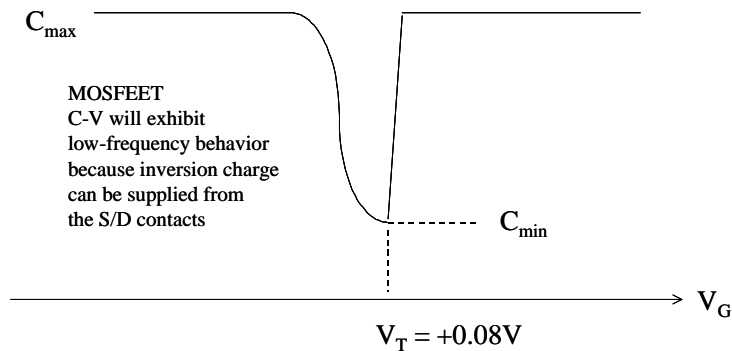
$$DV_T = q\phi / C_{ox} = 1.6 \times 10^{-19} \times 10^{12} / 1.725 \times 10^{-7} = 0.93\text{V}$$

$$\text{New } V_T = 0.93 + 0.07 = 1.0\text{V}$$

(f) $V_G - V_T = 5 - 1.01 < V_D = 5\text{V}$. Transistor is in saturation mode.

$$I_{Dsat} = k[(V_G - V_T)^2/2] = 398\text{mA}$$

(g)



$$(h) \ C_{max} = C_{ox} = 1.725 \cdot 10^{-7}\text{ F/cm}^2$$

$$C_{min} = C = \frac{1}{1/C_{ox} + x_{dmax}/\epsilon_s} = 2.88 \cdot 10^{-8}\text{ F/cm}^2$$