MOSFET I-V Analysis

N-MOSFET

- In general, inversion charge $Q_n \propto [V_G - V_T]$ decreases from Source toward Drain because channel potential $V_C$ increases.

\[ V_B = 0 \]
Approximate Analysis

Let $V_T$ defined to be threshold voltage at Source

$$I_D = W t \cdot (-q n v_{\text{drift}})$$

$$= W \cdot Q_n \cdot v_{\text{drift}}$$

Note: $I_D$ is constant for all positions along channel

$V_T (\text{average}) \sim V_T + \frac{V_{DS}}{2}$ [This is an approximation]

$Q_n (\text{average}) = C_{OX} \left( V_G - V_T (\text{average}) \right)$

$$= C_{OX} \left( V_G - V_T - \frac{V_{DS}}{2} \right)$$
With \( v_{\text{drift}} = -\mu_n E \approx \frac{\mu_n V_{DS}}{L} \)

\[
I_D = \mu \frac{W}{L} C_{OX} \left( V_G - V_T - \frac{V_{DS}}{2} \right) V_{DS}
\]
V_D saturation

Lateral E-field $\rightarrow \infty$
Electrons moves saturation velocity

$V_S=0$

Electrons move at saturation velocity

$Q_n=0$ at the drain

$V_{D\text{sat}}$ is defined to be the value of $V_D$ with $Q_n=0$ at drain.

From $Q_n = C_{ox} (V_G - V_T - V_D)$, we get $V_{D\text{sat}} = V_G - V_T$
Saturation Current

- saturation region:

\[ V_D \geq V_{Ds} = V_{GS} - V_T \]

\[ I_{Dsat} = \frac{W}{2L} C_{oxe} \mu_{eff} (V_{GS} - V_T)^2 \]
Pinch-Off & Channel-Length Modulation

$V_{GS} > V_T$:

$V_{DS} = V_{GS} - V_T$

$V_{DS} > V_{GS} - V_T$

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MOSFET I-V Characteristics Summary

For $V_D < V_{D_{sat}}$

$$I_D = \frac{\mu_n W}{L} C_{OX} \left( V_G - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

For $V_D > V_{D_{sat}}$

$$I_D = I_{D_{sat}} = \frac{\mu_n W}{2L} C_{OX} (V_G - V_T)^2$$

Note: $V_{D_{sat}} = V_G - V_T$
Mobility of inversion charge carriers

* Carrier will experience additional scattering at the Si/SiO₂ interface

* Channel mobility is lower than bulk mobility

\[ \mu_{(\text{effective})} \text{ is extracted from MOSFET I-V characteristics} \]

* Typically \(~0.5\) of \(\mu_{(\text{bulk})}\)
Parameter Extraction from MOSFET I-V

(A) $V_T$

For $V_D = V_G > V_T$

$V'_T$ at drain

$= V_{FB} + V_D + 2|\phi_p|$

$+ \frac{1}{C_{OX}} \sqrt{2\varepsilon_s qN_a (2|\phi_p| + V_D)}$

$\Rightarrow V_G - V'_T < 0$

$\Rightarrow$ Drains is at pinch-off

$\Rightarrow$ MOSFET is in saturation mode.
\[
\therefore I_D = I_{D_{Sat}} = k \frac{W}{L} (V_D - V_T)^2
\]

\[
\sqrt{I_D} \quad \mu_n C_{OX}
\]

\[
\text{slope} = \sqrt{\frac{kW}{L}}
\]

\[
V_T \quad V_D
\]

\[
V_G
\]
Alternative way to extract $V_T$

- Measure $I_D$ versus $V_G$ for a **fixed small** $V_{DS}$ (say <100mV)

$$I_D = \frac{\mu_n W}{L} C_{OX} \left( V_G - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

$$\approx \frac{\mu_n W}{L} C_{OX} (V_G - V_T) V_{DS}$$

The intercept of $I_D$ versus $V_G$ plot on $V_G$-axis is $V_T$. 
(B) Body Coefficient $\gamma$

$$\gamma \equiv \frac{V_T (\text{with } V_{SB} \neq 0) - V_T (\text{with } V_{SB} = 0)}{\sqrt{2|\phi_p| + |V_{SB}|} - \sqrt{2|\phi_p|}}$$

$$= \frac{\sqrt{2}\varepsilon_s qN_a}{C_{OX}}$$
\[ I_D = \mu_n \frac{W}{L} C_{OX} \left( V_G - V_T - \frac{V_D}{2} \right) V_D \]

\[ \frac{\partial I_D}{\partial V_D} = \mu_n C_{OX} \frac{W}{L} (V_G - V_T) \text{ for small } V_D \]
(D) Transconductance $g_m$

$$g_m = \frac{\partial I_D}{\partial V_G} \bigg|_{\text{fixed } V_D}$$

(a) For $V_{DS} < V_{Dsat}$

$$I_D = \frac{\mu_n W}{L} C_{OX} \left( V_G - V_T - \frac{V_{DS}}{2} \right)V_{DS}$$

$$\therefore \frac{\partial I_D}{\partial V_G} = \mu_n C_{OX} \frac{W}{L} \cdot V_{DS} \quad [g_m \text{ varies with } V_{DS}]$$

(b) For $V_{DS} > V_{Dsat}$

$$I_D = I_{Dsat} = \frac{\mu_n W}{2L} C_{OX} \left( V_G - V_T \right)^2$$

$$\frac{\partial I_D}{\partial V_G} = \frac{\mu_n W}{L} C_{OX} \cdot (V_G - V_T) \quad [g_{msat} \text{ varies with } V_G]$$

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(E) Channel Modulation Parameter $\lambda$

\[ I_{D_{sat}} = \frac{k}{2} (V_G - V_T)^2 (1 + \lambda V_{DS}) \]

Typically $\lambda \sim 0.1 \text{ to } 0.01 \text{ (volt)}^{-1}$
Short Channel Effect on $V_T$

depletion charge controlled by gate.

definition of $V_T$ and $L$
\begin{align*}
L' &= L - 2x \\
&= L - 2\left[\sqrt{(X_j + W_o)^2 - W_o^2} - X_j\right] \\
&= L - 2X_j \left[\sqrt{1 + \frac{2W_o}{X_j}} - 1\right]
\end{align*}

Note: \( W_o \) is \( x_{\text{dmax}} \)

Same electric potential because of heavily doped n+
Area of gate charge distribution

\[
q \cdot N_a \cdot \frac{L + L_1}{2} \cdot W_o \cdot W
\]

\[
\frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{\text{area of actual distribution}}{\text{area of ideal distribution}}
\]

\[
= 1 - \frac{X_j}{L} \left[ \sqrt{1 + \frac{2W_o}{X_j}} - 1 \right] \equiv f
\]

“Yau Model” for short-channel effect.
• Implantation at low energy
• Small Dt.
• Minimize channeling and transient enhance diffusion

To make $f \rightarrow 1$

$X_j$ \downarrow

$W_o$ \downarrow

• Increase $N_a$
**Effect of $V_{DS}$ on $V_T$ Lowering**

Large $V_{DS}$ ⇒ Larger S/D depletion charge at the drain side
⇒ Smaller depletion region charge contributed by gate
⇒ $V_T$ starts to decrease at larger $L$

![Diagram of depletion layer and depletion charge contributed by gate](image-url)
Narrow Width Effect (related to W)

Ideal Depletion charge

parasitic charge which has to be created by gate bias

\[ \therefore V_T \text{ is larger than ideal analysis.} \]
Narrow Width Effect

Narrow Channel Effect
Small Geometry Effects Summary

Actual gate control charge

Ideal gate control charge

W

L