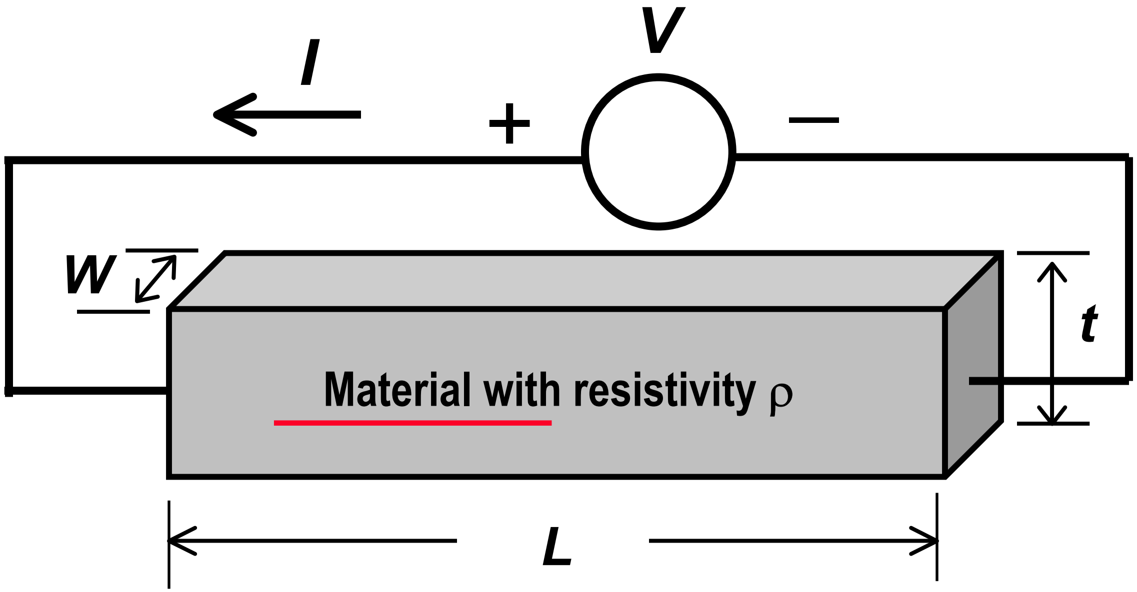
Electrical Resistance

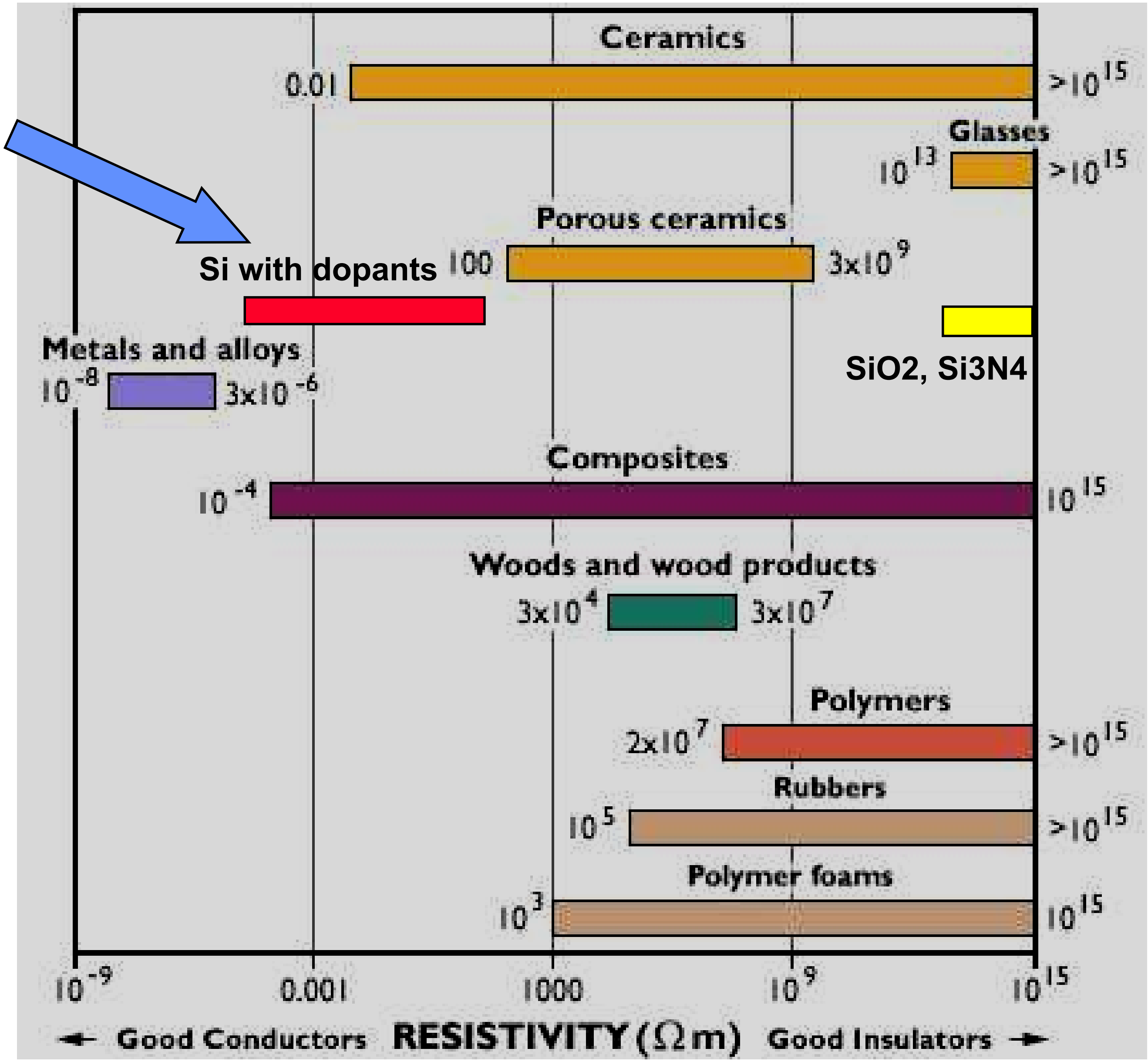


Resistance $$\boxed{R \equiv \frac{V}{I} = \rho \frac{L}{Wt}}$$ (Unit: ohms)

where ρ is the *electrical resistivity*

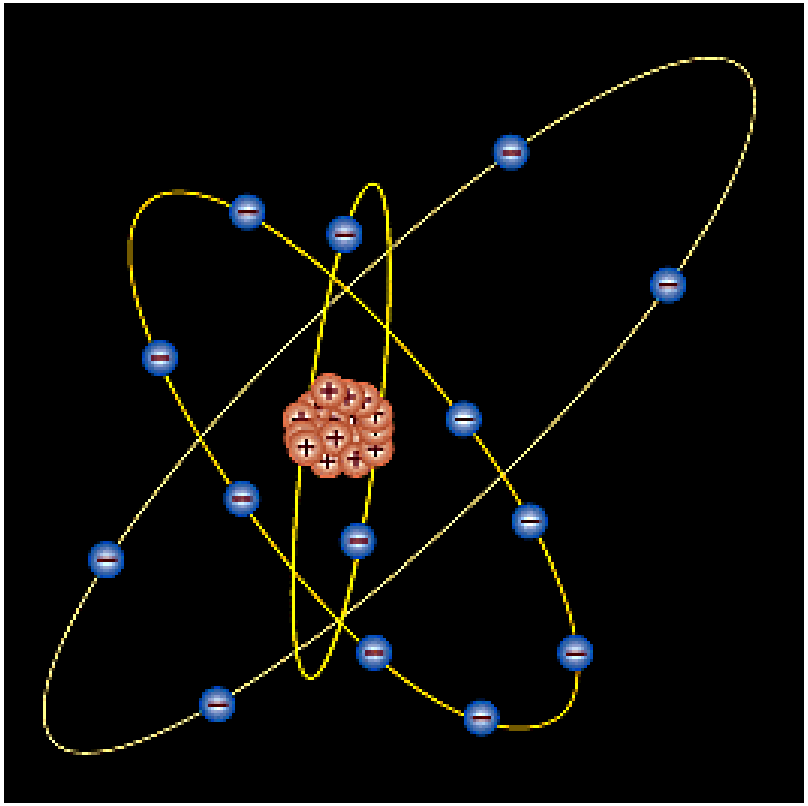
Resistivity Range of Materials

Adding parts/billion to parts/thousand of “dopants” to pure Si can change resistivity by 8 orders of magnitude !

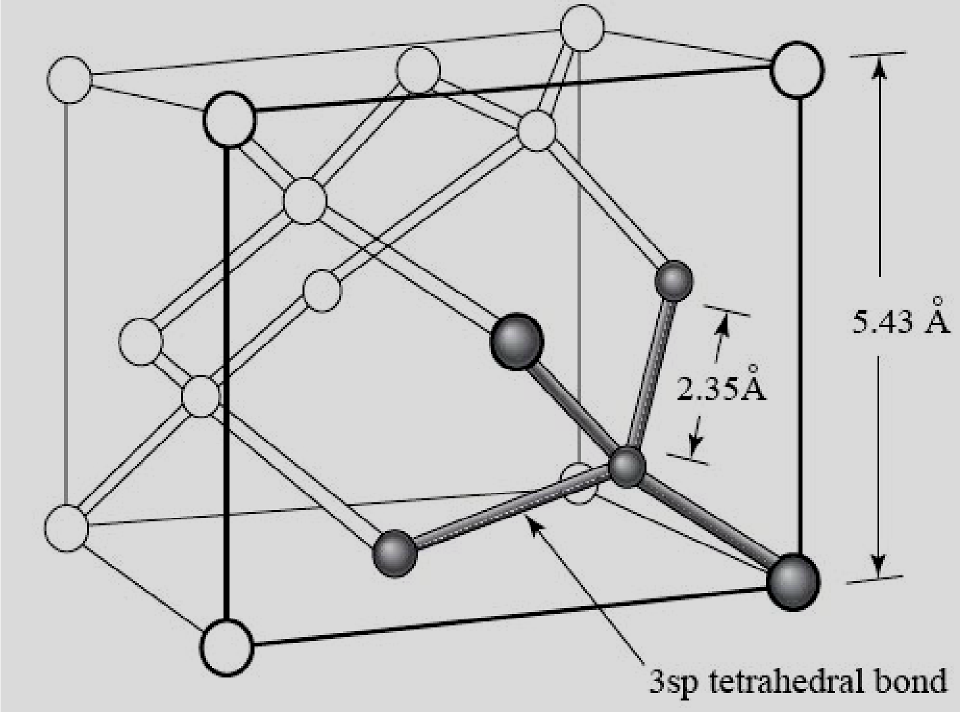


$$1\ \Omega\text{-m} = 100\ \Omega\text{-cm}$$

The Si Atom



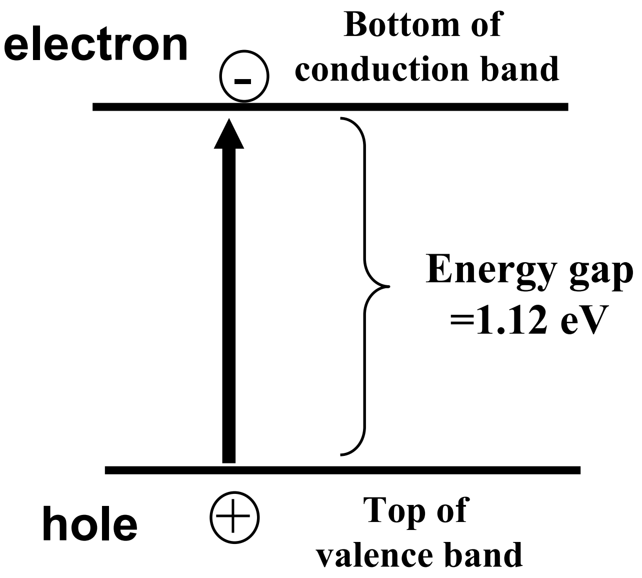
The Si Crystal

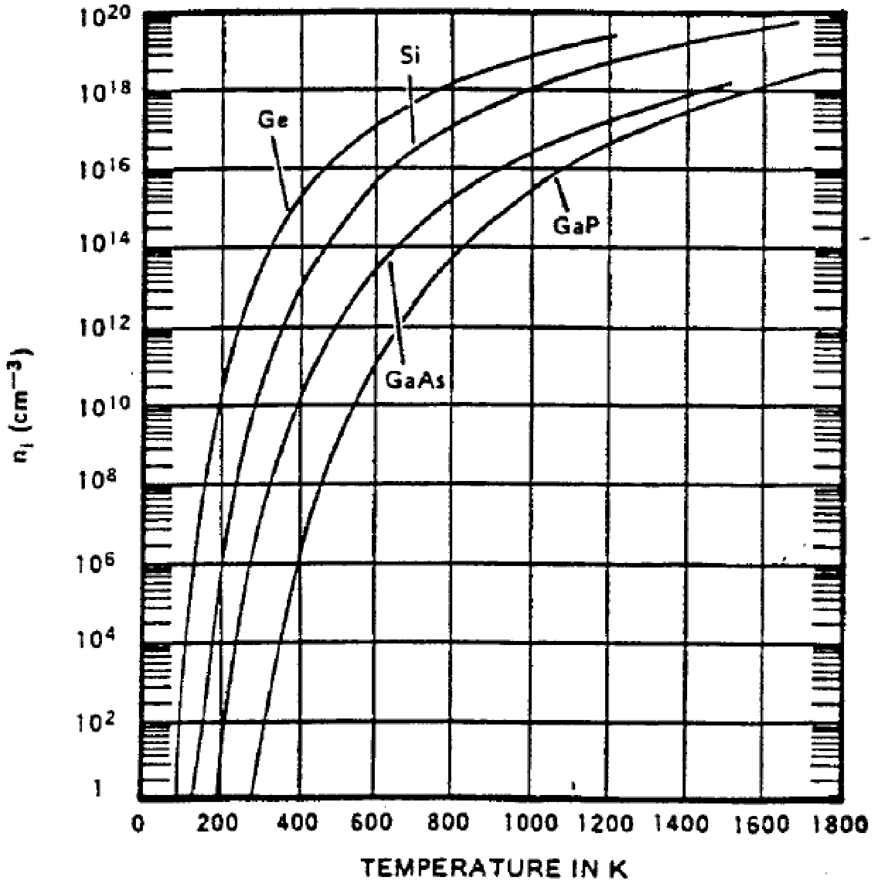


“diamond” structure

High-performance semiconductor devices require defect-free crystals

Carrier Concentrations of Intrinsic (undoped) Si





n (electron conc)
= p (hole conc)
= n_i

For Si

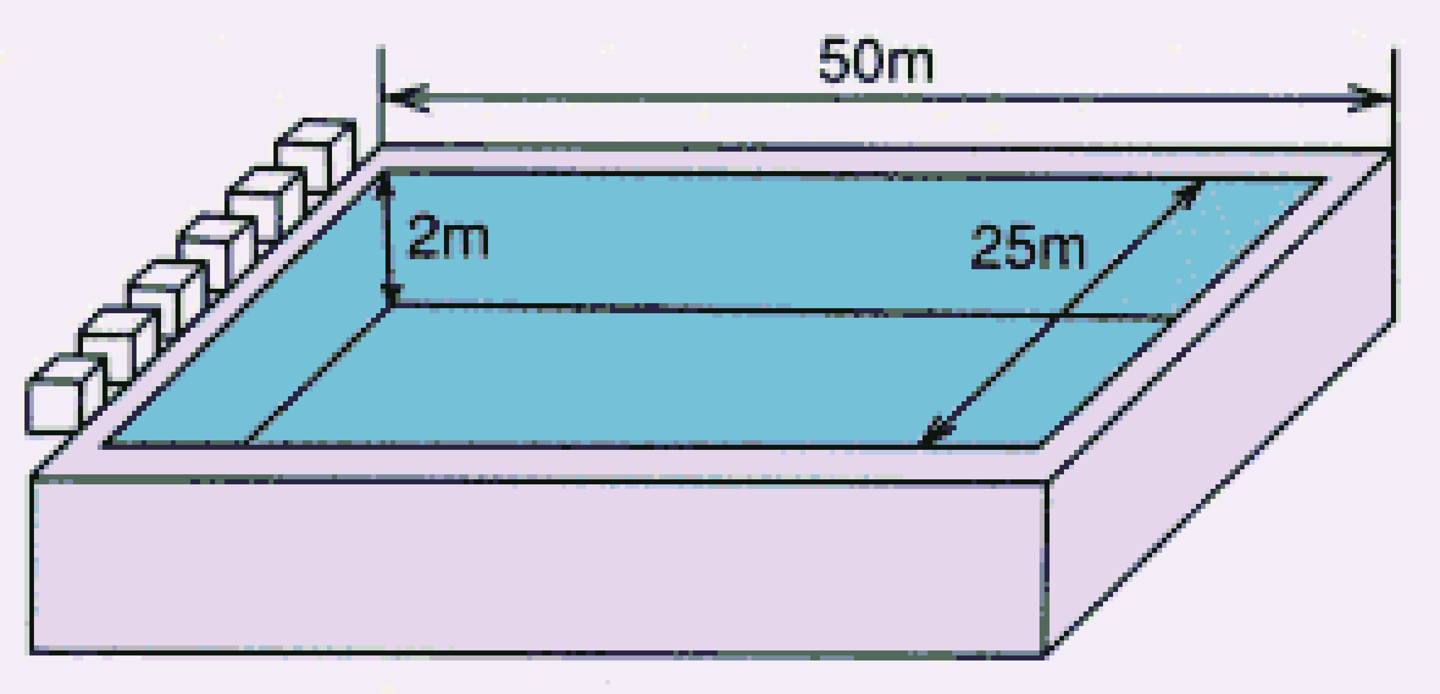
$$n_i = 3.9 \times 10^{16} T^{3/2} e^{-\frac{0.605eV}{kT}} /cm^3$$

$n_i \cong 1.45*10^{10}$ cm^{-3} at room temperature

Purity of Device-Grade Si wafer

99.999999999 % (so-called "eleven nines") !!

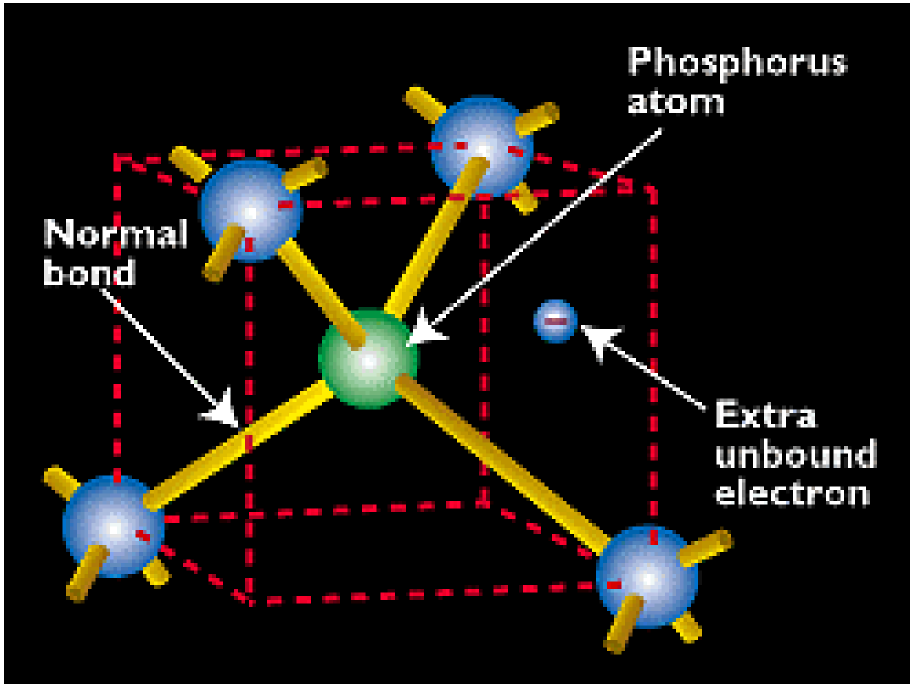
Maximum impurity allowed is equivalent to
1 mg of sugar dissolved in an Olympic-size swimming pool.



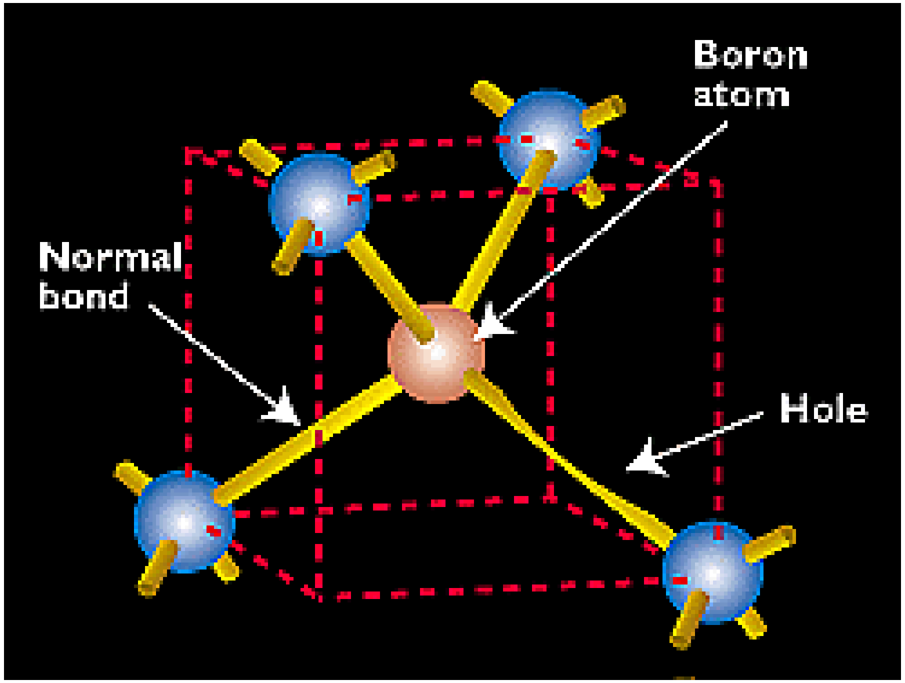
Dopants in Si

By <u>substituting</u> a Si atom with a special impurity atom (**Column V** or **Column III** element), a conduction electron or hole is created.

<u>Donors</u>: P, As, Sb



<u>Acceptors</u>: B, Al, Ga, In

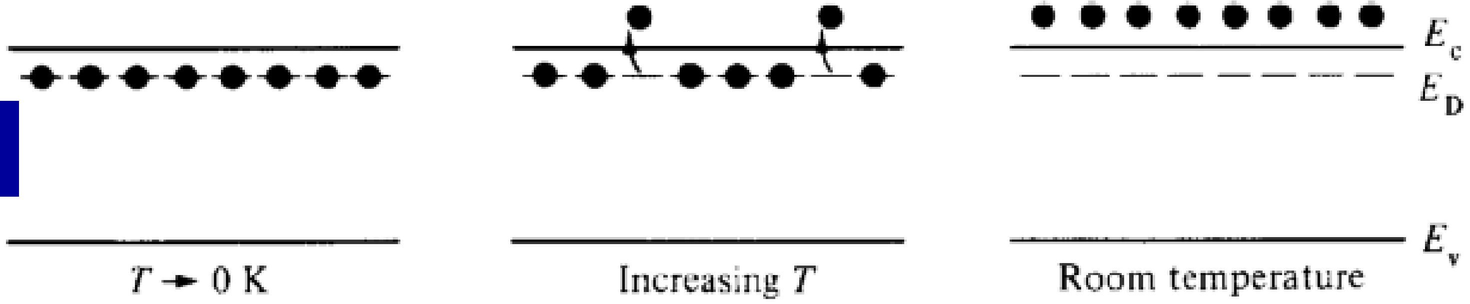


Energy Band Description of Electrons and Holes Contributed by Donors and Acceptors

E_C = bottom of conduction band
E_V = top of valence band
E_D = Donor energy level
E_A = Acceptor energy level

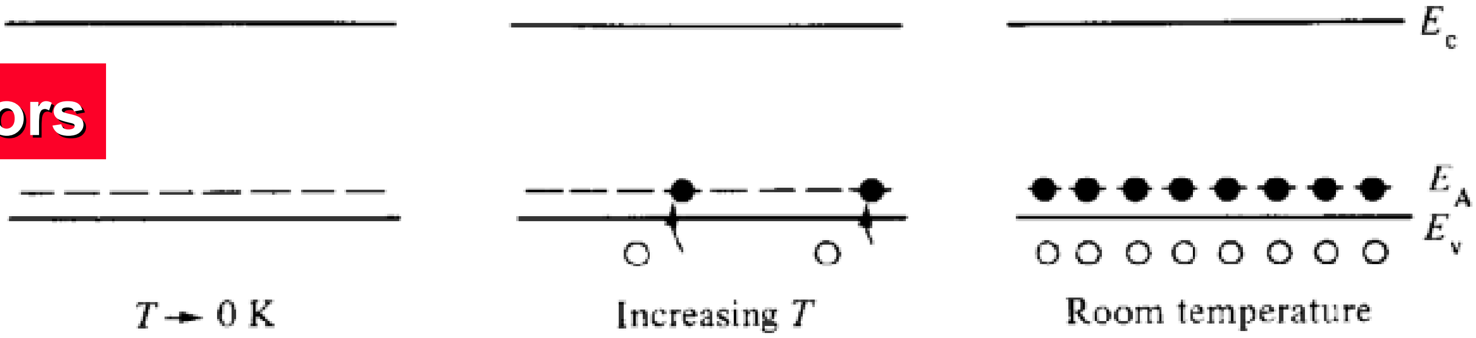
At room temperature, the dopants of interest are essentially fully ionized

Donors



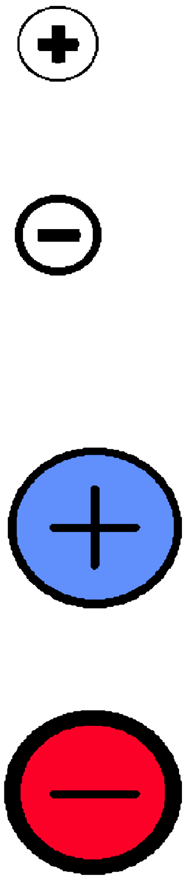
(a)

Acceptors



(b)

Semiconductor with both acceptors and donors has 4 kinds of charge carriers



Hole

Electron

Mobile;
they contribute to current flow with electric field is applied.

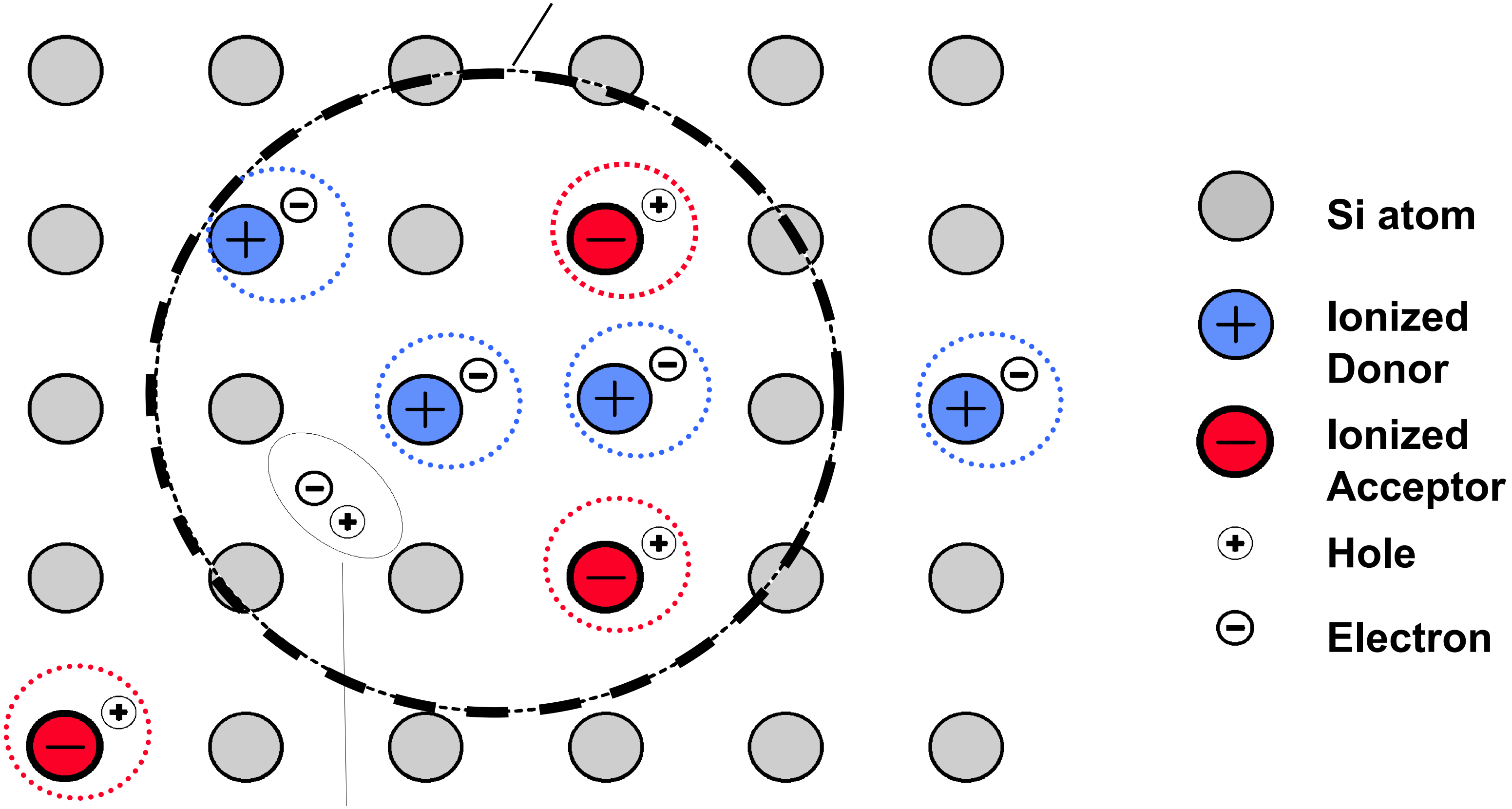
Ionized Donor

Ionized Acceptor

Immobile ; they DO NOT contribute to current flow with electric field is applied. However, they affect the local electric field

Charge Neutrality Condition

Valid for homogeneously doped semiconductor at thermal equilibrium



Even N_A is not equal to N_D, microscopic volume surrounding any position x has zero net charge

electron-hole pair due to transition from valence band to conduction band

How to Calculate Electron and Hole Concentrations for homogeneous Semiconductor

n: electron concentration (cm^{-3})

p : hole concentration (cm^{-3})

N_D: donor concentration (cm^{-3})

N_A: acceptor concentration (cm^{-3})

Assume completely ionized

1) Charge neutrality condition: $N_D + p = N_A + n$

2) At thermal equilibrium, $np = n_i^2$ ("Law of Mass Action")

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

Note: Carrier concentrations depend on *NET* dopant concentration (N_D - N_A) !

N-type and P-type Material

If $N_D \gg N_A$ (so that $N_D - N_A \gg n_i$):

$$n \cong N_D - N_A \quad \textbf{and} \quad p \cong \frac{n_i^2}{N_D - N_A}$$

***n* >> *p* → material is "n-type"**

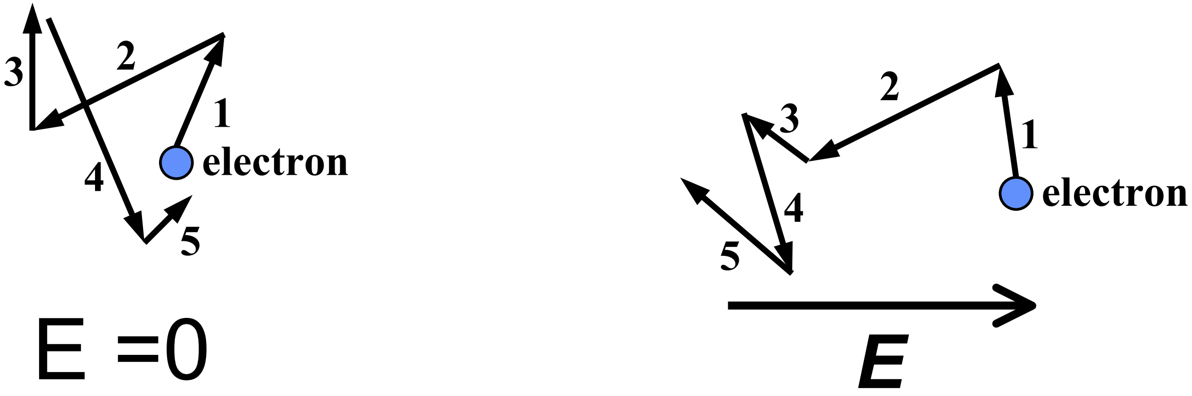
If $N_A \gg N_D$ (so that $N_A - N_D \gg n_i$):

$$p \cong N_A - N_D \quad \textbf{and} \quad n \cong \frac{n_i^2}{N_A - N_D}$$

***p* >> *n* → material is "p-type"**

Carrier Drift

- When an electric field is applied to a semiconductor, mobile carriers will be accelerated by the electrostatic force. This force superimposes on the random thermal motion of carriers:



E.g. Electrons *drift* in the direction opposite to the *E*-field

→ Current flows

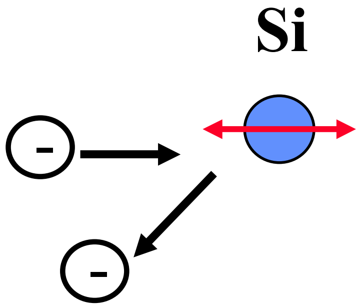
Average *drift velocity* = $|\boldsymbol{v}| = \mu E$

Carrier mobility

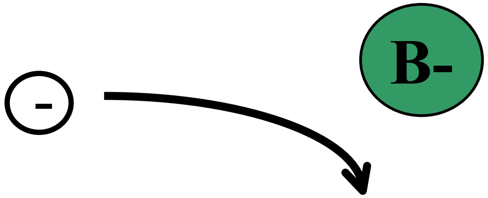
Carrier Mobility

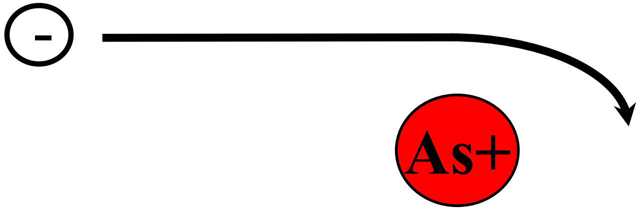
- Mobile carriers are always in random thermal motion. If no electric field is applied, the average current in any direction is zero.

- **Mobility is reduced by**
  - collisions with the vibrating atoms
    - “phonon scattering”



  - deflection by ionized impurity atoms

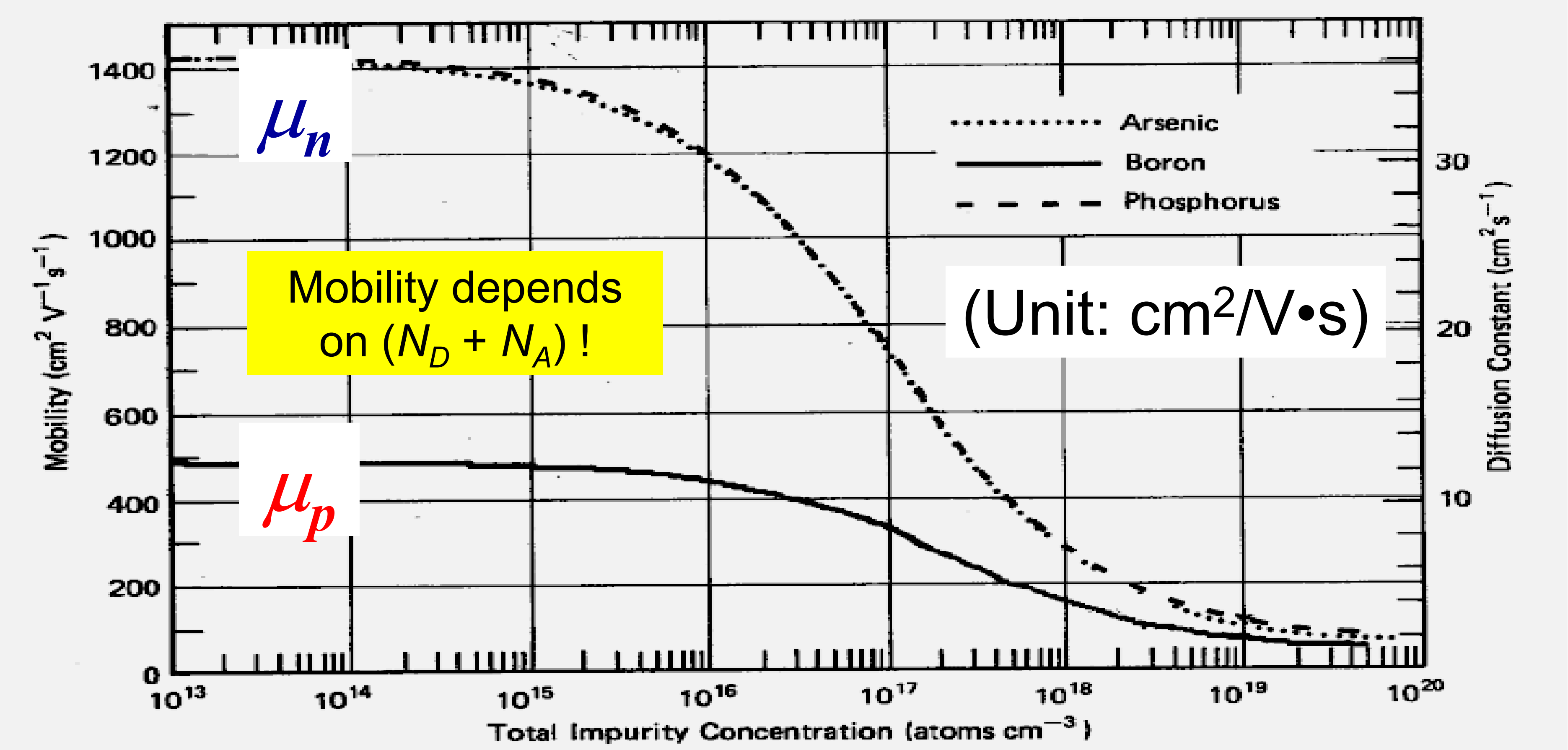




Carrier Mobility μ

Mobile charge-carrier drift velocity is proportional to applied *E*-field:

$$\boxed{|v| = \mu E}$$



Mobility depends on ($N_D + N_A$) !

(Unit: cm^2/V•s)

Electrical Conductivity σ

When an electric field is applied, current flows due to drift of mobile electrons and holes:

electron current density: $$J_n = (-q)nv_n = qn\mu_n E$$

hole current density: $$J_p = (+q)pv_p = qp\mu_p E$$

total current density: $$J = J_n + J_p = (qn\mu_n + qp\mu_p)E$$

$$J = \sigma E$$

conductivity $$\boxed{\sigma \equiv qn\mu_n + qp\mu_p}$$

Electrical Resistivity ρ

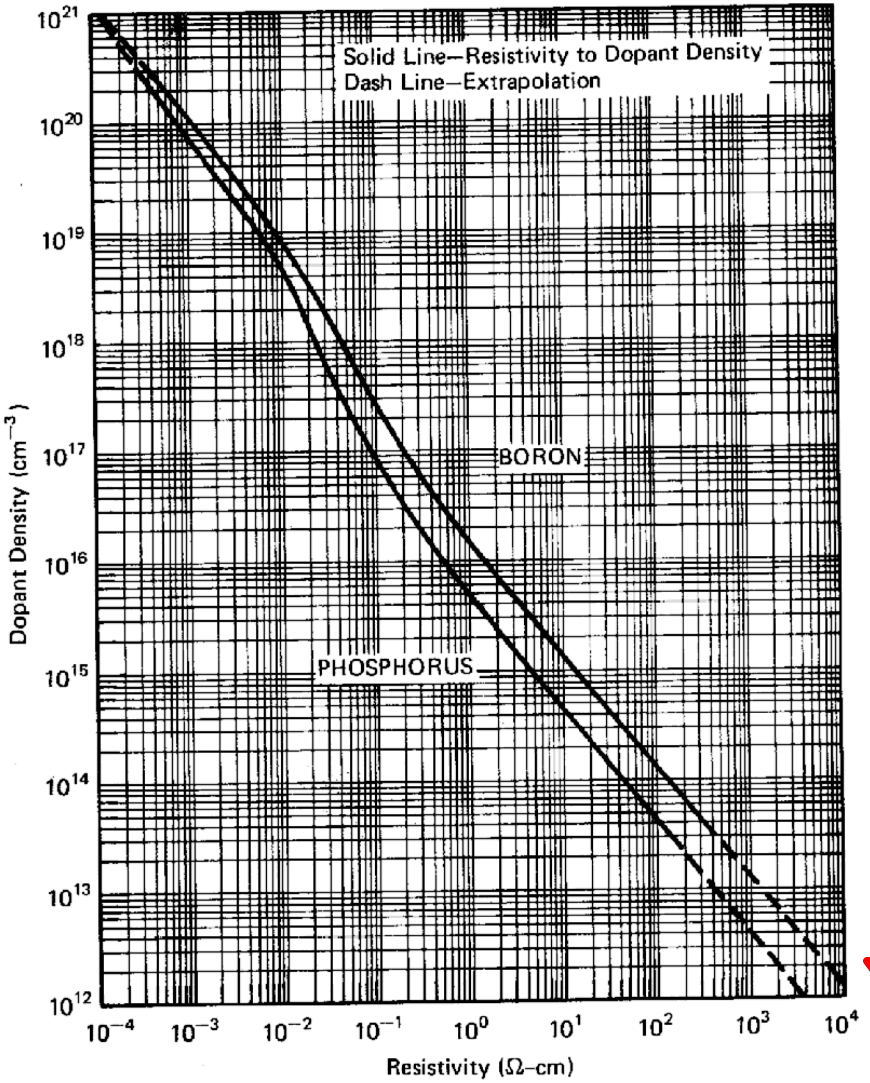


Figure 1.14 Dopant density versus resistivity at 23°C (296 K) for silicon doped with phosphorus and with boron. The curves can be used with little error to represent conditions at 300 K. [W. R.

$$\boxed{\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}}$$

$$\rho \cong \frac{1}{qn\mu_n}$$ for n-type

$$\rho \cong \frac{1}{qp\mu_p}$$ for p-type

(Unit: ohm-cm)

Note: This plot does not apply for compensated material!

Example Calculation

Consider a Si sample doped with $10^{16}/cm^3$ Boron. What is its **electrical resistivity**?

Answer:

$N_A = 10^{16}/cm^3$, $N_D = 0$ ($N_A >> N_D$ → **p-type**)

→ $p \approx 10^{16}/cm^3$ and $n \approx 10^4/cm^3$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qp\mu_p}$$

$$= \left[(1.6\times10^{-19})(10^{16})(450)\right]^{-1} = 1.4\,\Omega - \mathrm{cm}$$

From μ vs. ($N_A + N_D$) plot

Example: **Dopant Compensation**

Consider the same Si sample (with $10^{16}/\text{cm}^3$ Boron), doped ***additionally*** with $10^{17}/\text{cm}^3$ Arsenic. What is the new resistivity?

Answer:

$N_A = 10^{16}/\text{cm}^3$, $N_D = 10^{17}/\text{cm}^3$ ($N_D >> N_A$ → **n-type**)

→ $n \approx 9\text{x}10^{16}/\text{cm}^3$ and $p \approx 1.1\text{x}10^{3}/\text{cm}^3$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qn\mu_n}$$

$$= \left[(1.6\times10^{-19})(9\times10^{16})(600)\right]^{-1} = 0.12\,\Omega-\text{cm}$$

*** The sample is converted to n-type material by adding more donors than acceptors, and is said to be "compensated".**

Summary of Doping Terminology

intrinsic semiconductor: undoped semiconductor

extrinsic semiconductor: doped semiconductor

donor: impurity atom that increases the electron concentration

group V elements (P, As)in Si

acceptor: impurity atom that increases the hole concentration

group III elements (B, In) in Si

n-type material: semiconductor containing more electrons than holes

p-type material: semiconductor containing more holes than electrons

majority carrier: the most abundant mobile carrier in a semiconductor

minority carrier: the least abundant mobile carrier in a semiconductor

mobile carriers: Charge carriers that contribute to current flow when electric field is applied.

Sheet Resistance R_S

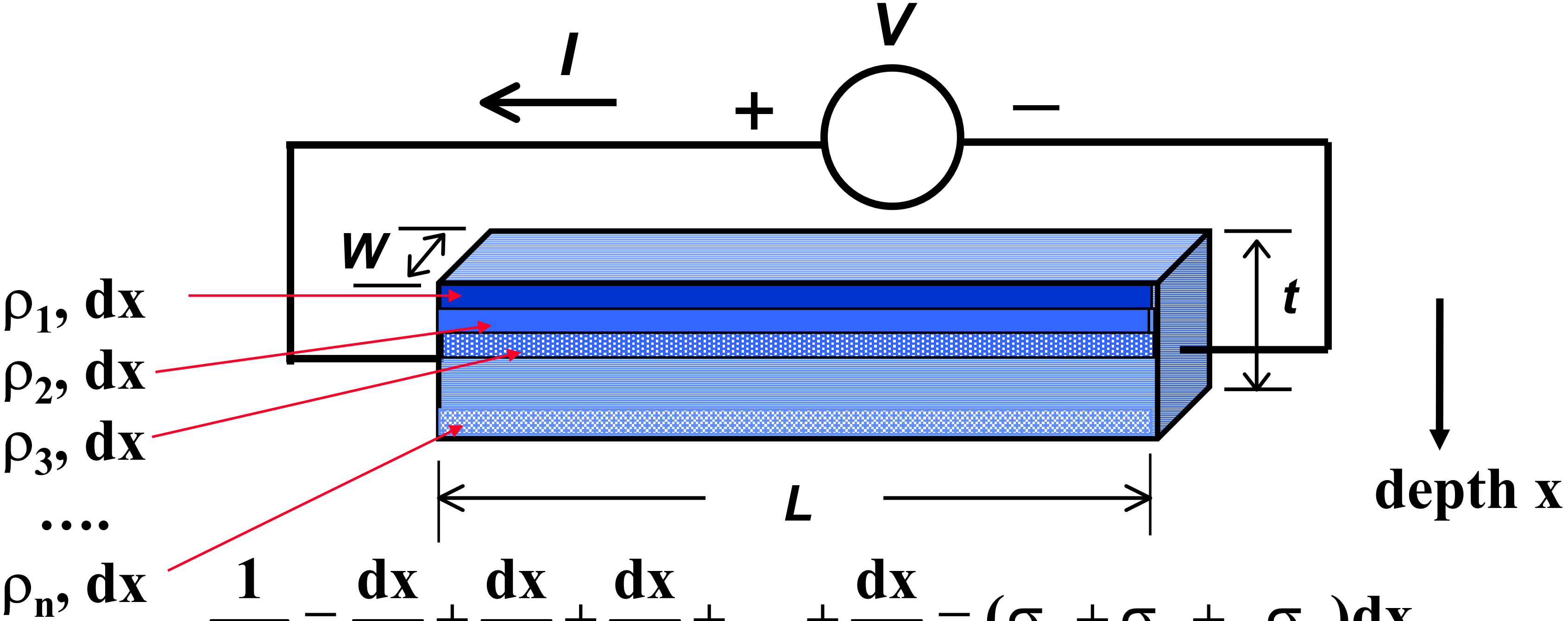
$$R = \rho \frac{L}{Wt} = R_s \frac{L}{W}$$

R_s is the resistance when $W = L$ (unit in ohms/square)

$$\mathbf{R_s \equiv \frac{\rho}{t}}$$ **if ρ is independent of depth x**

- The R_s value for a given layer (*e.g.* doped Si, metals) in an IC or MEMS technology is used
  - for design and layout of resistors
  - for estimating values of parasitic resistance in a device or circuit

R_S when $\rho(x)$ is function of depth x



$$\frac{1}{R_S} = \frac{dx}{\rho_1} + \frac{dx}{\rho_2} + \frac{dx}{\rho_3} + \ldots. + \frac{dx}{\rho_n} = (\sigma_1 + \sigma_2 + ..\sigma_n)dx$$

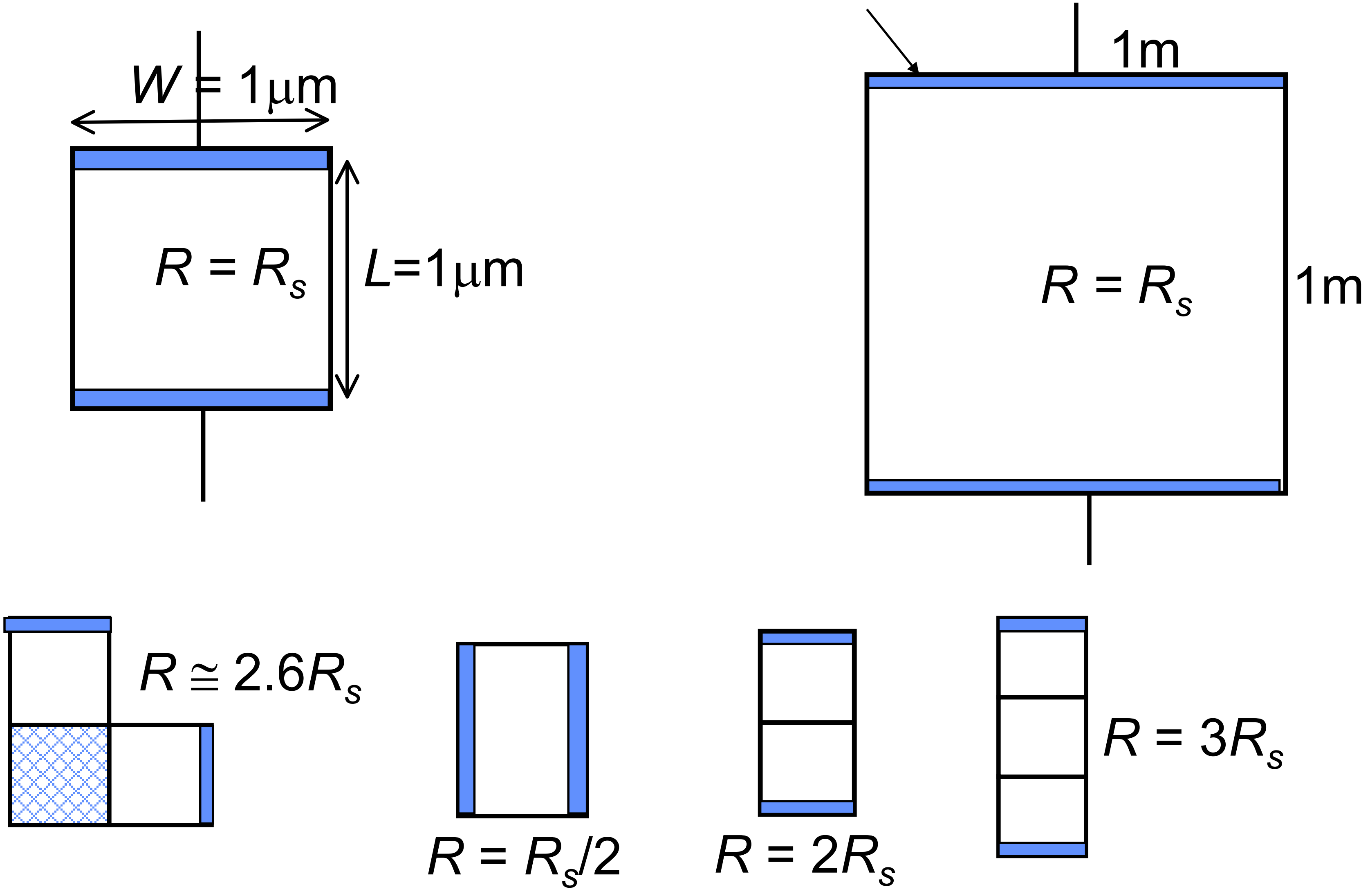
For a continuous σ(x) function: $R_S = \dfrac{1}{\int_0^t \sigma(x)dx}$

$$= \frac{1}{\int_0^t \left[q\mu_n(x)n(x) + q\mu_p(x)p(x)\right]dx}$$

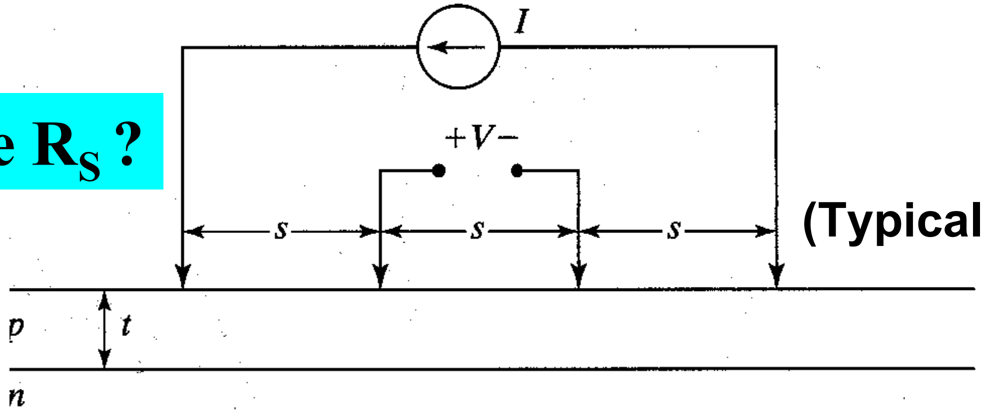
Electrical Resistance of Layout Patterns

(Unit of R_S: ohms/square)

Top View



How to measure R_S ?



(Typically, $s \approx$ 1 mm >>t)

- The ***Four-Point Probe*** is used to measure R_s
  - – 4 probes are arranged in-line with equal spacing ***s***
  - – 2 outer probes used to flow current ***I*** through the sample
  - – 2 inner probes are used to sense the resultant voltage drop ***V*** with a voltmeter

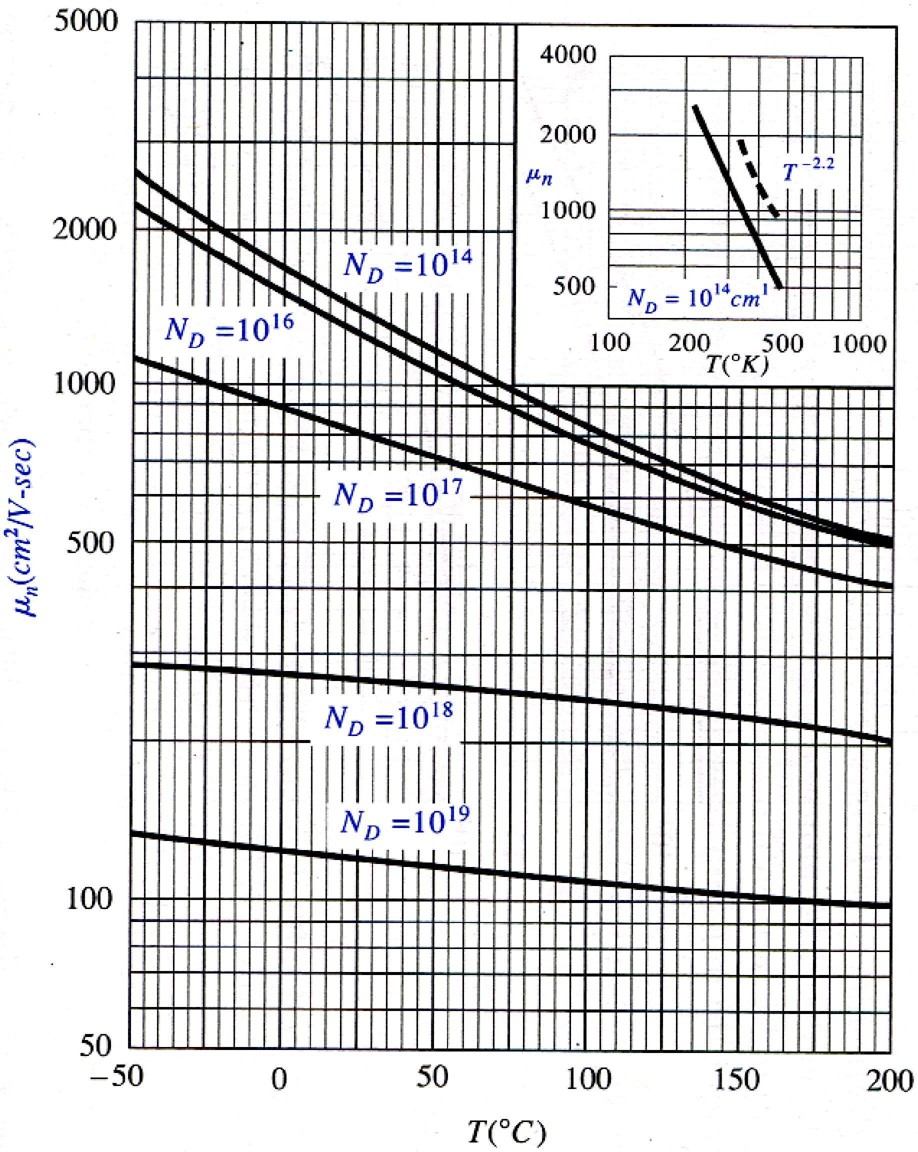
For a *thin* layer ($t \leq s/2$), $R_s = \dfrac{4.532V}{I}$

If ρ is known, then R_s measurement can be used to determine *t*

For derivation, see EE143 Lab Manual
http://www-inst.eecs.berkeley.edu/~ee143/fa05/lab/four_point_probe.pdf

For reference only

Electron mobility *vs.* T



Hole mobility *vs.* T