Lecture 6: Frequency Response Inspection Analysis

- Announcements:
  - Make-up lecture for next Tuesday
    - Friday: 6 p.m. in ???
  - Lecture Topics:
    - Amplifier Bode plot
    - Open Circuit Time Constant (OCTC) Analysis
    - Frequency Response Inspection Analysis
    - Frequency Response Examples

- Last Time:

---

**MOS Inspection Formulas w/ Substrate Grounded**

- $g_m \rightarrow g_{m+g_{mb}}$
- $r_{in} = \frac{g_m}{g_m+g_{mb}}$
- $B \rightarrow \infty$
- $R_G = \frac{1}{g_m+g_{mb}}$
- $R_s = \frac{1}{g_m+g_{mb}}$
- $R_d = R_b \left[ 1 + (g_m+g_{mb})R_b \right]$

---

**Effect of $g_{mb}$ (one more example)**

\[
\frac{N_d}{N_s} = \frac{-G_m R_s}{G_m \left( 1 + (g_m+g_{mb}) R_b \right)}
\]

\[
\frac{N_d}{N_s} = \frac{-G_m R_s}{G_m \left( 1 + (g_m+g_{mb}) R_b \right)}
\]

\[
\frac{N_d}{N_s} = \frac{-G_m R_s}{G_m \left( 1 + (g_m+g_{mb}) R_b \right)}
\]

Remark: When the substrate is tied to the source, $g_{mb} = 0$. 

---

Copyright © 2012 Regents of the University of California
Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

\[ A(s) = A_m F_L(s) F_H(s) \]

where:

- **A_m** is the midband gain.
- **F_L(s)** and **F_H(s)** are the low frequency shaping and high frequency shaping functions, respectively.

![Diagram](image)

We already found this using small-signal analysis.

### High Freq. Response Determination Using Open Clkt. Time Constant (OCTC) Analysis

In general:

\[ F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_H} s^{n_H}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}} \]

where:

- \( n_p > n_H \)
- \( n_p \) and \( n_H \) are the orders of the numerator and denominator polynomials, respectively.

From which:

\[ b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{p_{n_p}}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{p_i}} = \sum_{k=1}^{n_p} Z_{pk} \]

is the coefficient of the 1st order term.
Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

\[ \prod_{i=1}^{n_p} \frac{1}{Z_{pi}} = \sum_{j} C_j R_{jo} = \sum_{j} \frac{1}{Z_{jo}} \]

where \( C_j \) are capacitors in the H.F. ckt., i.e., small ones.

\( R_{jo} \) = driving pt. resistance seen between the terminals of \( C_j \) determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage source, open current source)
3. short all large (coupling/bypass) capacitors (i.e., >1µF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) \( \omega_H \ll \omega_{p1}, \ldots, \omega_{pmp} \)

(ii) \( F_H(s) \equiv \frac{1}{1 + \frac{s}{\omega_H}} \)

(iii) \( b_i \approx \frac{1}{\omega_H} \rightarrow \omega_H = \omega_{p1} \equiv \frac{1}{b_1} = \frac{1}{\sum_j C_j R_{jo}} \)

When there is no dominant pole, an approximate expression for \( \omega_H \) is:

\[ \omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \ldots + \frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \ldots \ldots}} \]

(just FYI)
Final $\omega_H$: $\omega_H = \frac{1}{\frac{1}{\tau_0} + \frac{1}{\tau_{\text{M}}}}$

Now, use Miller's Theorem

Final $\tau_0$:

$\tau_0 = (R_f R_R) R_f (C_{\text{M}} + C_R (1 + g_m R_f))$

Final $\tau_{\text{M}}$:

$\tau_{\text{M}} = \frac{C_{\text{M}}}{\frac{R_f + g_m R_f}{1 + g_m (\frac{1}{g_m R_f}})$

Final $\tau_2$:

$\tau_2 = \frac{C_{\text{M}}}{\frac{R_f + g_m R_f}{1 + g_m (\frac{1}{g_m R_f}})$

$\omega_H = \frac{1}{\tau_0 + \tau_{\text{M}} + \tau_2 + \tau_3}$

$\frac{V_{out}}{V_{in}} \approx \frac{1}{\frac{g_m}{g_m + \frac{1}{g_m}}}$
**EE 140: Analog Integrated Circuits**

**Lecture 6w: Frequency Response Inspection Analysis I**

---

**Mos Two-Stage Amplifier**

Step 1: Eliminate caps.

Step 2: Determine time constants.

1. \[ C_{gd1} + C_{gd1} (1 + gmR_1) \] \( \cong \) \( R_5 \)

2. \[ C_{gd1} + C_{gd1} + C_{gd2} \] \( \cong \) \( R_0 \)

3. \[ C_{sb2}(R_{s2})(1 + \frac{1}{gm+gm_b}) \] \( \cong \) \( R_{s2} \)

4. \[ C_{sr2} \]

\( \frac{R_{d1} + R_{s2}}{1 + (gm+gm_b)R_{s2}} \)

\( \omega_4 \) is quite high...

(See C+H)

\[ \omega_4 = \frac{1}{\omega_0 + \omega_2 + \omega_3 + \omega_{sr2}} \]

---

**Low Freq. Amplifier Response Using Short-Circuit Time Constant Analysis (SCTC)**

Recall:

\[ |A(j\omega)| \rightarrow \text{midband gain} \]

In general, for the low freq. response:

\[ P_1(s) = \frac{S^{n} + a_1S^{(n-1)} + \ldots}{S^{m} + a_2S^{(m-1)} + \ldots} \]

\( n \leq m \), \( \# \text{poles} = \# \text{zeros} \)

We can express the coefficient \( e_1 \) by:

\[ e_1 = \omega_p + \omega_{p2} + \ldots + \omega_{p1} \]

For the case of a dominant pole:

\( \# \text{poles} = \# \text{zeros} \)

\[ \frac{S}{S + \omega_L} \rightarrow e_1 \cong \omega_H = \omega_L \]

\[ \omega_L \cong e_1 = \frac{\sum_{j=1}^{n} \omega_{p_j}}{j \sum_{j=1}^{m} C_{ij} R_{ij}} = \frac{1}{C_{ij} R_{ij}} \]

where \( C_j \) \( \cong \) various large (>10nF) capacitors in the clkt. (e.g., the bypass caps.)

\( R_{sj} \) \( \cong \) driving point resistance seen between the terminals of \( C_j \) determined with:

1. all large capacitors short-circuited, except \( C_j \), which is replaced by its test voltage source for \( R \) determination.

---

Copyright © 2012 Regents of the University of California
2) All independent sources eliminated
   (i.e., short voltage source, open current source)

3) Open all H.F. capacitors (i.e., small caps
   in the pf range, or <1 nF)

Again, for the case where there are no dominant poles,

A reasonable approximation is:

\[ \omega_L \approx \sqrt{\omega_p^2 + \omega_c^2 - 2\omega_p \omega_c - 2\omega_c^2} \]