Lecture 20: Narrowbanding vs. Pole-Splitting

Announcements:
- Welcome back (from Spring Break)
- Lab#3 in progress
- HW#9 online

Lecture Topics:
- Review Last Lecture
- Review of Pole/Zero Plots
- Compensation
  - Narrowbanding
  - Pole-Splitting
- Hand back graded midterm & discuss grading

Last Time: (it's been a long while, so we'll quickly review the whole last lecture)

Stability & Compensation in Op Amps

In general, op amps are used in nag FB loops.
Reason:
1. Feedback set the biasing - no large coupling or bypass capacitors needed.
2. FB increases BW.
3. FB increases linearity of input range.
   (e.g., emitter degeneration is a type of FB)
4. Gain determined by external FB components - more accurate than op amp gain.
5. FB sets Ri and Re.
6. FB can improve temperature stability.

Diagram:
- Problem: any FB loop can become unstable under certain conditions. Need to compensate the instability.
- Ex. Non-Inverting Amplifier
- V₀ = a(s)Vₑ
- Vₑ = Vᵢᵣ - V_f
  \[ V_f = fV₀ \]
  \[ A(s) = \frac{V₀(s)}{Vᵢᵣ(s)} = \frac{a(s)}{1+a(s)f} \]
  \[ T(s) = a(s)f \]
- Closed Loop Voltage Gain
- Loop Transmission
- Instability occurs when A(s) → ∞.
  \[ A(s) \cdot \frac{a(s)}{1+a(s)f} \rightarrow \frac{a(s)}{1} \text{ will also go} \]
  \[ a(s)f = -1 \]
  \[ \text{unstable when denominator} = (-) \]
In General:
If \(|a(s)f| \geq 1\) when \(a(s)f = -(180^\circ)\), then unstable.

This is just a simplified form of the Nyquist Criterion.

**Stability of FB Op Amp Using a Single Pole Op Amp**

For a single pole op amp: \(a(s) = \frac{a_0}{1 - \frac{s}{f_1}}\), open loop transfer function.

Thus, closed loop \(X(s)\) for:

\[
\frac{a(s)}{1 + a(s)f} = \frac{a_0}{1 + a_0f} \frac{1}{1 - \frac{s}{f_1(1 + a_0f)}}
\]

Feedback \(f\) & \(f_1\) & \(f_1(1 + a_0f)\) are freq. shaping terms.

\(a_0 = \) closed loop dc gain \(\rightarrow (1 + a_0f) \approx af \times \text{smaller} \quad \text{than} \quad a_0\)

\(df\)

\(T_0 = af = \) loop gain (defined at \(dc\))

\(T(s) = a(s)f = \) loop transfer function
\(\text{(defined for small freqs.)}\)

**Bode Plot:**
- To use to determine \(a(s)f\)
- When \(a(s)f| \geq 1\) then determine stability.

\(1(a(s)f) = 1\) then determine stability.
Reminders:

1. For the case of a single-pole op amp, FB can never reach $\angle a(j\omega) = -180^\circ$ (90° is the limit).

2. Thus, an op amp FB clkt. w/o $f_c$ cont. and using a single-pole op amp is always stable!

But add a few non-dominant poles - then instability is possible!

Since now, $\angle a(j\omega)$ can reach $-180^\circ$!

Can best visualize this via a Bode plot.
For the more general case where \( P(s) \) has multiple poles:

\[ A(s) \text{ has the same additional poles} \]

\[ \text{i.e., at freq. } > \text{ freq of } P(s), \text{ the } A(\omega) \text{ curve just follows the } A(s) \text{ curve} \]

\[ A(\omega) \approx \frac{A_0}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})(1 - \frac{s}{p_3})} \]

It makes sense, because at freq. > freq of \( P(s) \), the loop transmission \( |A(s)|f < 1 \) — thus really isn’t much FB anymore.

**Definition:**

Phase Margin = \( 180° + \angle A(j\omega) \text{ at freq. where } |a(j\omega)|f = 1 \)

\[ \Rightarrow \text{Phase Margin must be } > 0° \text{ for stability} \]

\[ \text{For Stability: Phase Margin } > 0° \]

\[ \Rightarrow \text{But for safety, design for} \]

\[ \text{Phase Margin } \geq 45° \]

(Design Criterion for oscillation, use PM \( \geq 60° \).)

Can see this pretty clearly on a root-locus plot.

As the PM continues to shrink, the peak grows.

"When the peak gets sufficiently large, the circuit will oscillate at its freq. -> Instability!"
**Definition:**

\[ \text{Gain Margin} = \left| T(j\omega) \right| \text{ in } \text{dB} \text{ at freq. where } \angle T(j\omega) = -180^\circ \]

For stability: \[ \text{Gain Margin} < 0 \text{dB} \]

**Comparison of Op Amps**

To compare, need the distance between \( p_1 \) and \( p_2 \) to be large enough to encompass the largest desired loop gain.

\[ \left| a(j\omega) \right| [\text{dB}] \]

Slope = -20 dB/dec

\[ 20 \log \left| \frac{P_2}{P_1} \right| = 20 \log \left| \frac{P_{\text{Max}}}{P_1} \right| \]

\[ \frac{P_{\text{Max}}}{P_1} = \frac{1}{10} \rightarrow \frac{P_2}{P_1} = \frac{1}{10} \frac{\text{Tomax}}{P_1} \]

\[ \text{For stability at } \pm 45^\circ \]

**Two Ways to Compensate:**

1. **Narrowbanding**
2. **Pole-Splitting**

\[ \text{Narrowbanding: } \text{dominant} = 0' \]

Introduce a pole \( p_0 \) so that is sufficient separation between \( p_0 \) and \( p_1 \), which becomes the "new \( p_2 \)"