HW5 EE140

1. a) $R_{OUT} = r_{o1} + r_{o2} + g_{m1} r_{o1} r_{o2}$

The "turn on" voltage is when the two right transistors are in saturation region.

It's set by V_{G4}

$$V_{G4} = V_{G3} + V_T + V_{DSAT}$$
$$= 2(V_T + V_{DSAT})$$

$$V_{S2} = V_{G2} - V_T - V_{DSAT}$$
$$= V_{G4} - V_T - V_{DSAT}$$
$$= V_T + V_{DSAT}$$

$$V_{OUT} = V_{D2} = V_{S2} + V_{DSAT} \quad (EOS)$$
$$= V_T + 2V_{DSAT} = V_{min} \text{ (turn on voltage)}$$

To find V_{DSAT} :

$$M3\&4: \frac{1}{2} k' \left(\frac{W}{L}\right) V_{DSAT}^2 \cdot (1 + \lambda(V_{DSAT} + V_T)) = 1\ mA$$

$\therefore V_{DSAT} = 0.30423\ V \approx .304\ V$

$\therefore V_{min} = 1.108\ V$

$$r_{o1} = \frac{1 + \lambda V_{DS1}}{\lambda I} = \frac{1 + \lambda(V_T + V_{DSAT})}{\lambda I} \doteq 10.8\ k\Omega$$

$$r_{o2} = \frac{1 + \lambda V_{DS2}}{\lambda I} = \frac{1 + \lambda(V_{DSAT} + V_T)}{\lambda I} \doteq 10.8\ k\Omega$$

$$g_{m1} = \frac{2I}{V_{DSAT}(1 + \lambda V_{DS})} \doteq 6.1\ mS$$

$\therefore R_{OUT} = g_{m1} r_{o1} r_{o2} + r_{o1} + r_{o2}$
$\doteq 732\ k\Omega$

Note: if you guys use $r_o = \frac{1}{\lambda I}$, $g_m = \frac{2I}{V_{DSAT}}$,

you will end up with $R_{OUT} \doteq 678\ k\Omega$

(b) see SPICE deck and plots.

2. a) VBN sets the source voltage of M2 and the drain voltage of M1, so VBN determines M1 in saturation region or not.

$V_{D1} = V_{DSAT}$ (EOS of M1)

$V_{BN,min} = V_{D1} + V_T + V_{DSAT}$
$= 2V_{DSAT} + V_T$
$= 1.108\ V$

Turn on voltage makes sure M1 & M2 are in saturation region.

$V_{o,min} = V_{DSAT1} + V_{DSAT2}$
$= 2V_{DSAT}$
$= 0.608\ V$

$R_{OUT} = r_{o1} + r_{o2} + g_{m1} r_{o1} r_{o2}$ the same as prob. 1

b) see SPICE deck and plots.

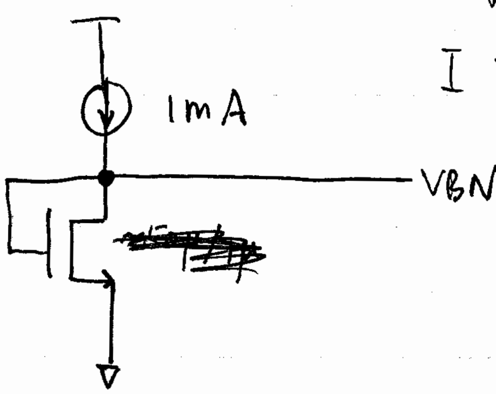
$r_o = 1/[I(OUT2) - I(OUT)]$ use expression builder.

c) cascode mirror w/ VBN is best.

Simulation should give very close resules.

	R_{OUT}	$V_{o,min}$
Cascode Mirror	$732\,k\Omega$	$1.108V$
Cascode Mirror w/ VBN	$732\,k\Omega$	$0.608V$

3.



$$V_{DSAT}' = 2V_{DSAT}$$

$$I = \frac{1}{2}k'\left(\frac{W}{L}\right)(2V_{DSAT})^2\left(1+\lambda(V_T+2V_{DSAT})\right)$$

$$\Rightarrow \left(\frac{W}{L}\right) = 24.35$$

To verify, put this into SPICE and it works fine.

4. Assume $g_m r_o \gg 1$.

a) $$\frac{v_o}{v_i} = -(g_{m1} r_{o1} r_{o2} \,\|\, R_L)\, g_{m1} \qquad \text{(cascode)}$$

$$\frac{v_c}{v_i} = -g_{m1} \cdot R_{OUT}$$

$$= -g_{m1}\left(r_o \,\|\, \frac{R_L}{R_L \| r_o} \cdot \frac{1}{g_{m2}}\right) \qquad \text{(CS)}$$

$$\frac{v_o}{v_{BN}} = \text{CS w/ source degeneration}$$

$$\frac{v_o}{v_{BN}} = R_{OUT} \cdot G_m$$

$$= \left(\left[r_{o1} + r_{o2}(1+g_{m2}r_{o1}) \right] \parallel R_L \right) \cdot \frac{-g_{m2}}{1+g_{m2}r_{o1}}$$

$$\frac{v_c}{v_{BN}} = \frac{g_{m2}}{1+\frac{R_L}{r_o}} \cdot \left(r_{o1} \parallel \frac{R_L \cdot 1/g_m}{R_L \parallel r_{o2}} \right) \qquad CD.$$

b) $R_L = r_o$. $\quad g_m = \frac{2I}{V_{OSAT}} \doteq 6.3\ mS,\ r_o \doteq 10k\Omega$

$$\therefore \frac{v_o}{v_i} = -g_m (r_o \parallel g_m r_o^2) = -g_m r_o = -63$$

$$\frac{v_c}{v_i} = -g_m \left(r_o \parallel \frac{r_o}{r_o \parallel r_o} \frac{1}{g_m} \right)$$

$$\approx -g_m \left(r_o \parallel \frac{2}{g_m} \right) \approx -g_m \cdot \frac{2}{g_m} = -2.$$

$$\frac{v_o}{v_{BN}} \approx r_o \cdot \frac{-g_{m2}}{1+g_{m2}r_{o1}} \approx -1$$

$$\frac{v_c}{v_{BN}} = \frac{g_m}{1+\frac{r_o}{r_o}} \cdot \left(r_o \parallel \frac{r_o}{r_o \parallel r_o} \frac{1}{g_m} \right) \approx 1$$

Note: Assume $g_m r_o \gg 1$ approximation.

c) $R_L = g_m r_o^2$

$$\frac{v_o}{v_i} = -g_m (g_m r_o^2 \parallel g_m r_o^2) = -\frac{1}{2} g_m^2 r_o^2 = -1984.5$$

$$\frac{v_c}{v_i} = -g_m \cdot \left(r_o \parallel \frac{g_m r_o^2 / g_m}{r_o} \right) = -\frac{1}{2} g_m r_o = -31.5$$

$$\frac{v_o}{v_{BN}} = \frac{g_m r_o^2}{2} \cdot \frac{-g_m}{1+g_m r_o} \doteq -\frac{1}{2} g_m r_o = -31.5$$

$$\frac{v_c}{v_{BN}} = \frac{g_m}{1+g_m r_o} \cdot (r_o \| r_o) \doteq \frac{1}{r_o} \cdot \frac{r_o}{2} \doteq \frac{1}{2}$$

5. open end design problem, no solution.