EE130

3-11-03
Prof. King

SUMMARY NOTES FOR P-N JUNCTIONS

1) Terminology

$N_A \equiv$ net dopant concentration on p-type side

$N_D \equiv$ net dopant concentration on n-type side

2) pn Junction Electrostatics

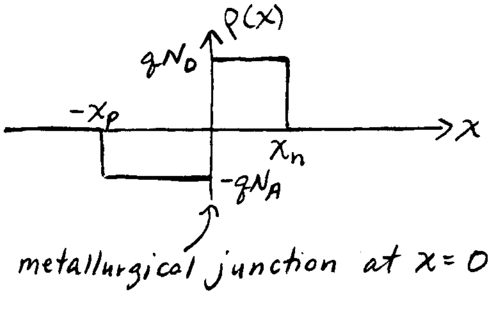
A region depleted of mobile charge carriers exists in the vicinity of the metallurgical junction, due to mobile charge carrier redistribution (holes diffusing into the n-type region, and electrons diffusing into the p-type region, initially)

This region extends a distance x_p into the p-type side, and a distance x_n into the n-type side.

"Depletion Approximation": $n, p << |N_D - N_A|$ in depletion region

$$\text{charge density } \rho(x) = \begin{cases} -qN_A & -x_p \leq x < 0 \\ qN_D & 0 < x \leq x_n \\ 0 & x < -x_p \text{ and } x > x_n \end{cases}$$

For a step junction, the charge-density distribution looks like



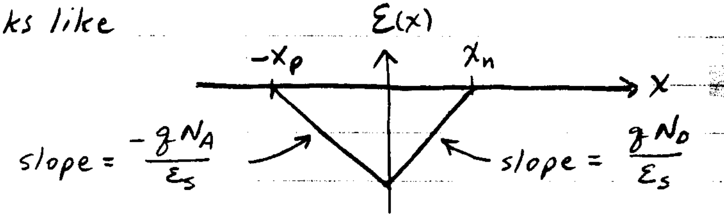
Note that $x_p N_A = x_n N_D$ for charge neutrality

$\Rightarrow$ The depletion region extends further into the side which is more lightly doped.

The charge distribution in the depletion region gives rise to a non-zero electric field distribution, which can be obtained using Poisson's Equation: $\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_s}$

$\rightarrow \mathcal{E} = \int \frac{\rho}{\varepsilon_s} dx$

For a step junction, the electric-field distribution looks like



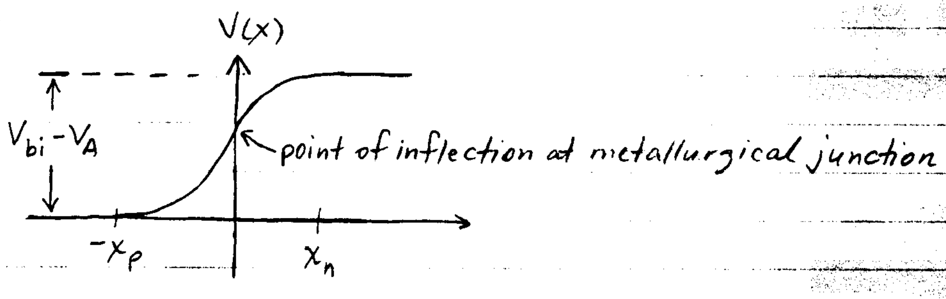
Note: $\mathcal{E} \approx 0$ in the quasi-neutral p-type and n-type regions ($x < -x_p$, $x > x_n$)

The electric field serves to counteract the diffusion of mobile charge carriers.

→ At equilibrium ($V_A = 0$), there is no net current flow across the junction.

The electric potential distribution is obtained by integrating the electric field distribution ($V = -\int \mathcal{E} dx$)

For a step junction, the electric-potential distribution looks like



Note that the area underneath the $\mathcal{E}(x)$ curve is equal to the voltage dropped across the depletion region, and that most of this voltage is dropped across the more lightly doped side.

In equilibrium ($V_A = 0$), the total voltage dropped across the depletion region is equal to the built-in potential V_{bi}, which is due to the work-function difference between the p-type and n-type regions:

$$qV_{bi} = (E_i - E_F)_{p\text{-side}} + (E_F - E_i)_{n\text{-side}}$$

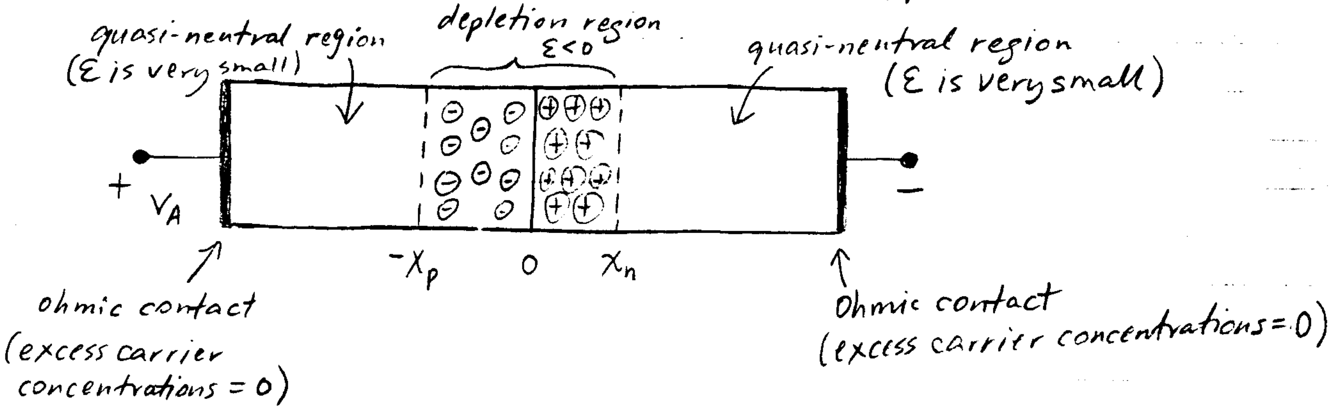
$$\text{where} \quad (E_i - E_F)_{p\text{-side}} = \begin{cases} E_G/2 & \ldots \text{ degenerate p-side} \\ kT \ln(N_A/n_i) & \ldots \text{ non-degenerate p-side} \end{cases}$$

$$(E_F - E_i)_{n\text{-side}} = \begin{cases} E_G/2 & \ldots \text{ degenerate n-side} \\ kT \ln(N_D/n_i) & \ldots \text{ nondegenerate n-side} \end{cases}$$

If both sides are non-degenerately doped, $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$

When an external bias voltage $V_A \neq 0$ is applied, the total voltage dropped across the junction is $V_{bi} - V_A$.

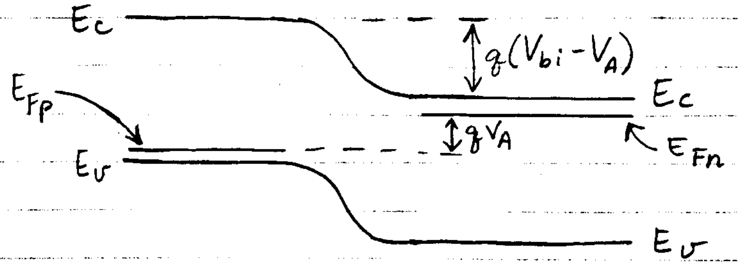
V_A is defined to be positive when the voltage applied to the p-side is higher than the voltage applied to the n-side:



Note that a positive V_A serves to counteract the built-in potential V_{bi}. (This is why the total voltage dropped across the junction is reduced by V_A.)
A negative V_A serves to reinforce the built-in potential.

$V_A > 0$: "forward bias"

Total voltage dropped across the junction is reduced:

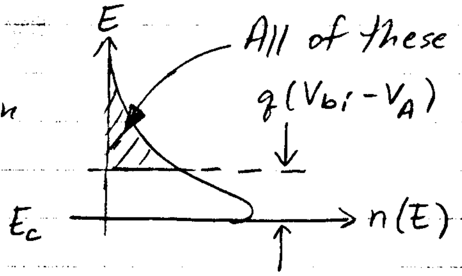


⇒ The potential barrier to mobile charge carrier diffusion across the junction is reduced by V_A.

Carriers with kinetic energy greater than $q(V_{bi}-V_A)$ can diffuse across the junction.

Since the carrier distributions in the valence band and in the conduction band decay exponentially with increasing carrier kinetic energy, the number of carriers with kinetic energy greater than $q(V_{bi}-V_A)$ increases exponentially with increasing V_A:

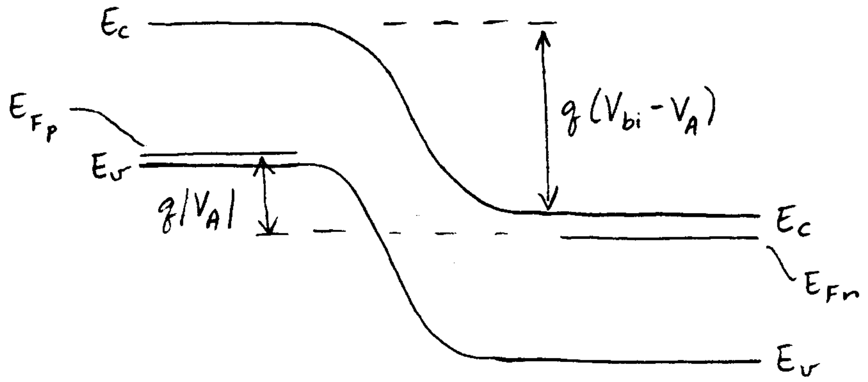
conduction band:



All of these electrons can diffuse across the junction (number increases exponentially with decreasing $V_{bi}-V_A$)

$V_A < 0$: "reverse bias"

Total voltage dropped across the junction is increased:



$\Rightarrow$ Very few mobile charge carriers can diffuse across the junction. Minority carriers which happen to diffuse into the depletion region are swept across the junction ("collected" into the quasi-neutral regions), constituting the only significant (negative) current which flows across the junction.

The width of the depletion region is changed accordingly:

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_A)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

Note that W increases with reverse bias, and decreases with forward bias.

For a <u>one-sided junction,</u> $W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_A)}{qN}}$

where N = net dopant concentration on lightly-doped side

The peak $\mathcal{E}$-field is $|\mathcal{E}(x=0)| = \sqrt{\frac{2qN(V_{bi}-V_A)}{\varepsilon_s}}$

The junction will breakdown under reverse bias if the peak electric field exceeds a critical value $\mathcal{E}_{crit}$:

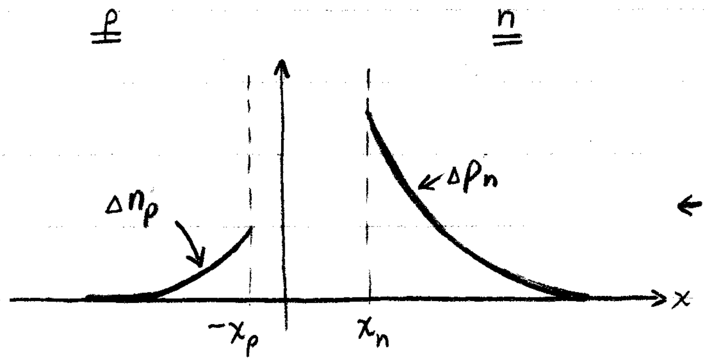
$$\sqrt{\frac{2qN(V_{bi}-V_A)}{\varepsilon_s}} > \mathcal{E}_{crit} \quad \leftarrow \text{one-sided junction}$$

This corresponds to a breakdown voltage V_{BR}:

$$V_{BR} = \frac{\varepsilon_s \mathcal{E}_{crit}^2}{2qN} - V_{bi} \quad \leftarrow \text{one-sided junction}$$

Note that V_{BR} decreases with increasing doping concentration.

3) EXCESS MINORITY CARRIER PROFILES IN A P-N JUNCTION



excess minority-carrie
profiles in a long-bas
diode under forward bi

Depletion-Edge Boundary Values:

$$\Delta n_p(-x_p) = n_{po}\left(e^{qV_a/kT}-1\right)$$

$$\Delta p_N(x_n) = p_{NO}\left(e^{qV_a/kT}-1\right)$$

Recall the "Law of the Junction"

$$pn = n_i^2 e^{qV_a/kT}$$

throughout depletion region

n_{po}, p_{NO} are equilibrium minority-carrier densities:

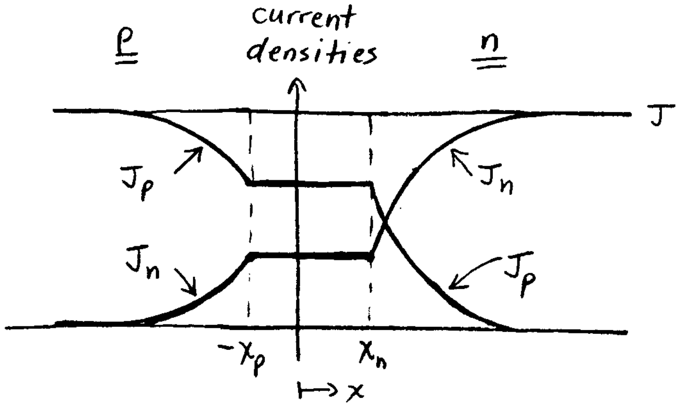
$$n_{po} = \frac{n_i^2}{N_a}\quad , \quad p_{NO} = \frac{n_i^2}{N_d}$$

$n_p(x)$ and $p_N(x)$ are found by solving the minority-carrier diffusion equations in the quasi-neutral regions:

$$\Delta n_p(x) = n_{po}\left(e^{qV_a/kT}-1\right)e^{+(x+x_p)/L_n} \qquad \text{p-side} \quad (x < -x_p)$$

$$\Delta p_N(x) = p_{NO}\left(e^{qV_a/kT}-1\right)e^{-(x-x_n)/L_p} \qquad \text{n-side} \quad (x > x_n)$$

4) P-N DIODE I-V CHARACTERISTICS



← components of current in a long-base diode under moderate forward bias

$$J = J_n + J_p = J_{n\,diff} + J_{n\,drift} + J_{p\,diff} + J_{p\,drift}$$

$J_{diff} \propto$ <u>gradient</u> of carrier concentration

$J_{drift} \propto$ <u>magnitude</u> of carrier concentration

Minority-carrier concentrations are very low

$\Rightarrow$ neglect J_{drift} for minority carriers:

p-region: $J_n = J_{n\,diff} = q D_n \frac{dn_p}{dx} = q D_n \frac{d\Delta n_p}{dx}$

n-region: $J_p = J_{p\,diff} = -q D_p \frac{dp_n}{dx} = -q D_p \frac{d\Delta p_n}{dx}$

Assuming no thermal R-G in depletion region,

$J_p(-x_p) = J_p(x_n)$ (also: $J_n(x_n) = J_n(-x_p)$)

Since total current flowing in diode is constant,

$$J = J_n(-x_p) + J_p(-x_p) = J_n(-x_p) + J_p(x_n)$$

Long-Base Diode:

$$\Delta n_p(x) = \frac{n_i^2}{N_a}\left(e^{qV_a/kT}-1\right)e^{(x+x_p)/L_n}$$

$$\Delta p_N(x) = \frac{n_i^2}{N_d}\left(e^{qV_a/kT}-1\right)e^{-(x-x_n)/L_p}$$

$$J_n(-x_p) = qD_n\left[\frac{n_i^2}{N_a}\left(e^{qV_a/kT}-1\right)\left(\frac{1}{L_n}\right)\right]$$

$$J_p(x_n) = -qD_p\left[\frac{n_i^2}{N_d}\left(e^{qV_a/kT}-1\right)\left(-\frac{1}{L_p}\right)\right]$$

$$J = qn_i^2\left[\frac{D_n}{L_nN_a}+\frac{D_p}{L_pN_d}\right]\left(e^{qV_a/kT}-1\right) = J_o\left(e^{qV_a/kT}-1\right)$$

Short-Base Diode:

$$J = qn_i^2\left[\frac{D_n}{W_p'N_a}+\frac{D_p}{W_N'N_d}\right]\left(e^{qV_a/kT}-1\right) = J_o\left(e^{qV_a/kT}-1\right)$$

where W_p' = width of quasi-neutral p-type region $(W_p - x_p)$

W_N' = width of quasi-neutral n-type region $(W_N - x_n)$

Note that diode current is dominated by the term associated with the more lightly doped side:

$$p^+n \text{ diode}: \quad J_o \simeq \begin{cases} qn_i^2\left(\frac{D_p}{L_pN_d}\right) & \text{long-base diode} \\ qn_i^2\left(\frac{D_p}{W_N'N_d}\right) & \text{short-base diode} \end{cases}$$

$$n^+p \text{ diode}: \quad J_o \simeq \begin{cases} qn_i^2\left(\frac{D_n}{L_nN_a}\right) & \text{long-base diode} \\ qn_i^2\left(\frac{D_n}{W_p'N_a}\right) & \text{short-base diode} \end{cases}$$

i.e.: The current flowing across the junction consists primarily of carriers injected from the more heavily doped side of the junction.

$$J_n(-x_p) \sim {n_i^2}/{N_a} \quad ; \quad J_p(x_n) \sim {n_i^2}/{N_d}$$

p^+n diode $(N_a >> N_d)$: $J \simeq J_p(x_n)$

n^+p diode $(N_d >> N_a)$: $J \simeq J_n(-x_p)$

NOTE: The excess majority-carrier concentration profiles are the same as the excess minority-carrier concentration profiles, to maintain charge neutrality.

i.e.: $\Delta p_P(x) = \Delta n_P(x)$ p-side

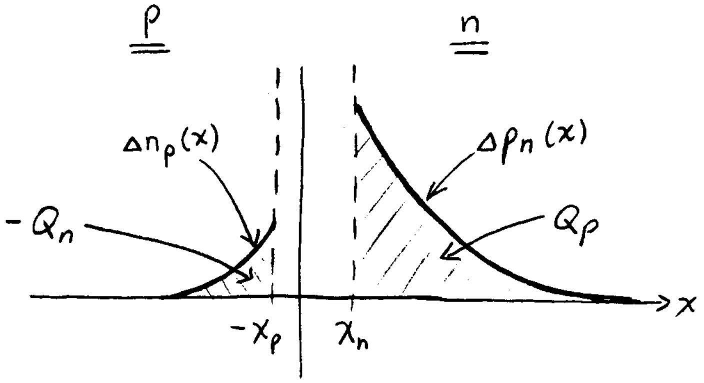
$\Delta n_N(x) = \Delta p_N(x)$ n-side

⇒ Majority-carrier current will have a diffusion component (contributing current in the direction opposite to the drift component):

$$J_p(x) = \underset{\text{negative, increasing exponentially in magnitude toward junction}}{J_{Pdiff}} + J_{Pdrift} \qquad \text{p-side}$$

$$J_n(x) = \underset{\text{negative, increasing exponentially in magnitude}}{J_{ndiff}} + J_{ndrift} \qquad \text{n-side}$$

3) MINORITY CARRIER CHARGE STORAGE IN P-N DIODE



Note that the minority-carrier charge stored in the diode is dominated by the component on the lightly doped side:

p^+n diode: $Q_p \gg Q_n$

n^+p diode: $Q_n \gg Q_p$

minority-carrier recombination lifetimes

$$Q_p = q\int_{x_n}^{W_N} \Delta p_n(x)\,dx = q\,\frac{D_p \tau_p}{L_p}\,\frac{n_i^2}{N_d}\left(e^{qV_a/kT}-1\right) = J_p(x_n)\,\tau_p$$

$$Q_n = -q\int_{-x_p}^{W_P} \Delta n_p(x)\,dx = -q\,\frac{D_n \tau_n}{L_n}\,\frac{n_i^2}{N_a}\left(e^{qV_a/kT}-1\right) = -J_n(-x_p)\,\tau_n$$

<u>Charge-control model</u>:

Currents supply charge to maintain the excess minority-carrier profiles. (This charge is consumed by recombination.)

$$J_p(x_n) = \frac{Q_p}{\tau_p}$$

$$J_n(-x_p) = \frac{-Q_n}{\tau_n}$$

(steady-state conditions)

☆ For short-base diode: replace τ_p, τ_n with <u>transit time</u>

$\tau_{tr} = \frac{W_N'^2}{2D_p}$ for holes

$\tau_{tr} = \frac{W_P'^2}{2D_n}$ for electrons

4) P-N DIODE DIFFUSION CAPACITANCE

$$C_d = \left| \frac{dQ}{dV_a} \right| \qquad (F/cm^2)$$

For a one-sided diode, we can consider the excess minority-carrier charge stored on the lightly doped side primarily:

p^+n diode: $$C_d = \frac{q Q_{po}}{kT} e^{qV_a/kT}$$

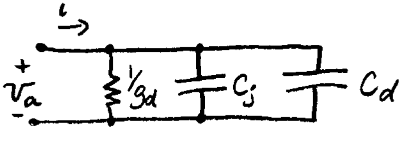
$$Q_{po} = \begin{cases} q p_{NO} L_p & \text{for long-base diode} \\ q p_{NO} \left(\frac{W_B'}{2}\right) & \text{for short-base diode} \end{cases}$$

- C_d is negligible under reverse bias (Q is very small)
- C_d increases exponentially with V_a under forward bias

5) P-N DIODE CONDUCTANCE

$$g_d \triangleq \frac{dJ}{dV_a} = \frac{q}{kT} J_o e^{qV_a/kT} = \frac{q}{kT}(J + J_o) \simeq \frac{q}{kT} J$$

diode small-signal equivalent circuit:



6) P-N JUNCTION DIODE : DEVIATIONS FROM THE IDEAL

a) Reverse Bias ($V_a < 0$)

Carrier concentrations are reduced below equilibrium values in the depletion region:

$$pn = n_i^2 e^{qV_a/kT} < n_i^2$$

=> there will be net generation in the depletion region

The generated carriers will be swept out of the depletion region due to the electric field:

electrons → n-type region
holes → p-type region

Thus, the net generation results in additional negative diode current:

$$I_g \approx -\frac{qAn_i}{2\tau_0} W_{dep}$$

where $\tau_0 = \frac{1}{2}\left[\tau_p e^{(E_t-E_i)/kT} + \tau_n e^{(E_i-E_t)/kT}\right]$

"recombination lifetime"

b) Forward Bias ($V_a > 0$)

Carrier concentrations are increased above equilibrium values in the depletion region:

$$pn = n_i^2 e^{qV_a/kT} > n_i^2$$

$\Rightarrow$ there will be <u>net recombination</u> in the depletion region

The net recombination results in an additional positive diode current:

$$I_r \simeq \frac{qAn_iW_{dep}}{2\tau_0} \exp\left(\frac{qV_a}{2kT}\right)$$

- This additional component of current is <u>significant at low current densities</u> ($V_a < 0.375$ V in Si diodes) because $(I_r/I) \sim \exp\left(-\frac{qV_a}{2kT}\right)$

- Since the diode diffusion current is proportional to n_i^2 (from ideal diode equation) and the recombination current is proportional to n_i, $(I_r/I) \sim \frac{1}{n_i}$

  $\Rightarrow$ I_r is negligible in Ge diodes, and
  I_r is negligible in Si diodes at elevated temperature.

At high forward biases, additional effects become important:

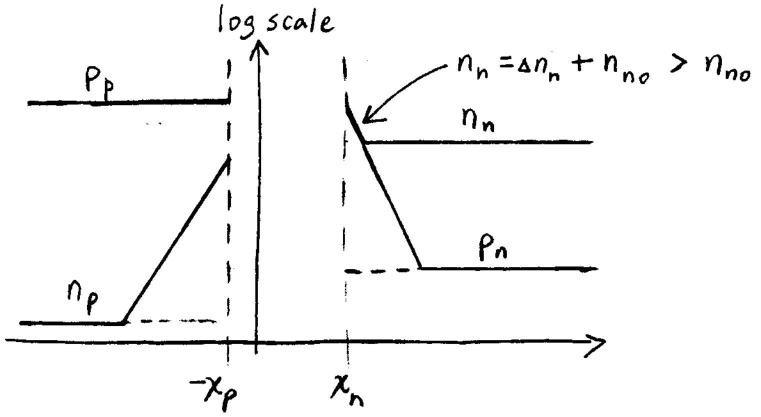
i) Series Resistance

- Voltage applied across the junction is reduced from V_a, due to the voltage drops across the quasi-neutral regions of the diode:

$$V_j = V_a - IR_s$$

ii) High-level injection (HLI)

Carrier concentration profiles under HLI:



$(n_{no} = N_d)$

- As V_a is increased, excess minority and majority carrier concentrations will increase.

$$\Delta p_P = \Delta n_P = n_{Po}\left(e^{qV_a/kT} - 1\right) \qquad p\text{-side}$$

$$\Delta n_N = \Delta p_N = p_{No}\left(e^{qV_a/kT} - 1\right) \qquad n\text{-side}$$

- For moderate V_a:

$$\Delta p_P \ll p_{Po} \quad \Rightarrow \quad p_P \simeq p_{Po} = N_a \qquad p\text{-side}$$

$$\Delta n_N \ll n_{No} \quad \Rightarrow \quad n_N \simeq n_{No} = N_d \qquad n\text{-side}$$

i.e. majority-carrier concentrations are not significantly increased above equilibrium values

→ "low-level injection" conditions prevail

- For large V_a:

As V_a increases, the side of the junction which is more lightly doped will reach HLI condition:

$$\Delta n_N \gtrsim n_{No} \quad \Rightarrow \quad n_N > n_{No}$$

i.e. The majority-carrier concentration is significantly increased above equilibrium value.

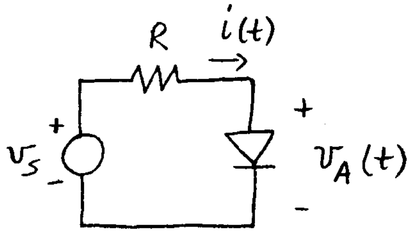
⇒ significant gradient in majority-carrier profile

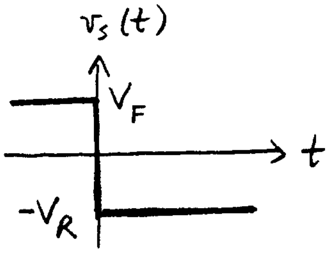
Net effect of HLI is to decrease effectiveness of V_a in increasing diode current:

$$I \sim e^{qV_a/2kT}$$

at high biases
(V_a within a few tenths of a Volt of ϕ_i)

7) DIODE TRANSIENT RESPONSE : TURN-OFF

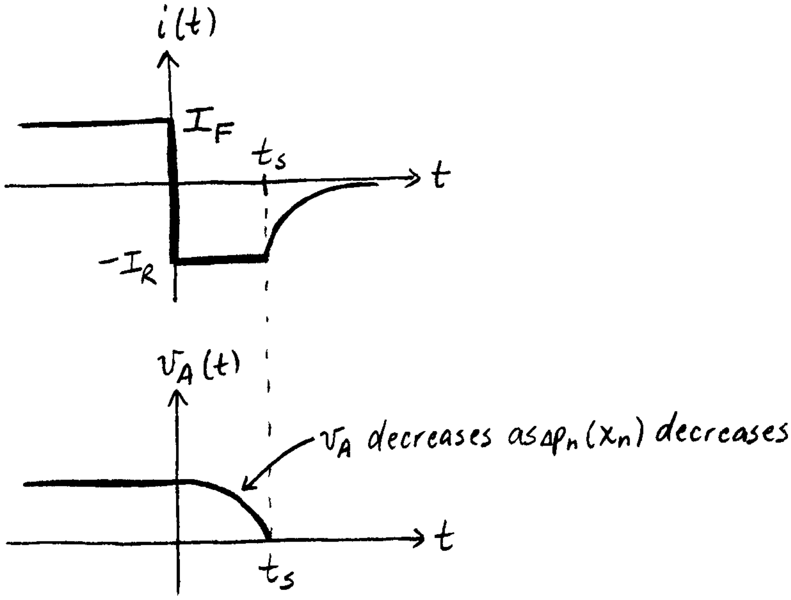




$I_F \simeq \frac{V_F}{R}$

$I_R \simeq \frac{V_R}{R}$

t_s : "storage delay time"



v_A decreases as $\Delta p_n(x_n)$ decreases

In order to turn the diode off, the excess minority carriers (injected into the quasi-neutral regions under forward bias) must be removed:

- by net carrier flow, and/or
- by recombination

Consider a one-sided (p^+n) diode : $\frac{dQ_p}{dt} = i - \frac{Q_p}{\tau_p}$

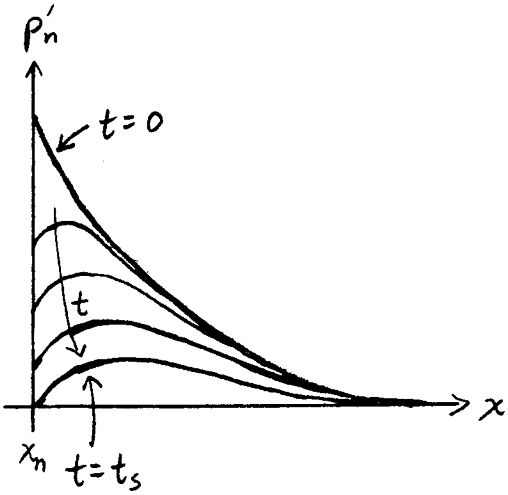
differential equation for Q_p

Decay of stored hole charge inside a p^+n diode:

★ Remember that the depletion-edge boundary value of Δp_n is related to v_A:

$\Delta p_n(x_n) = p_{no}(e^{qV_a/kT}-1)$

$\Rightarrow$ if $\Delta p_n(x_n) > 0$, $V_a > 0$

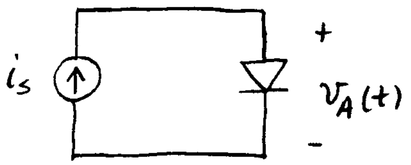


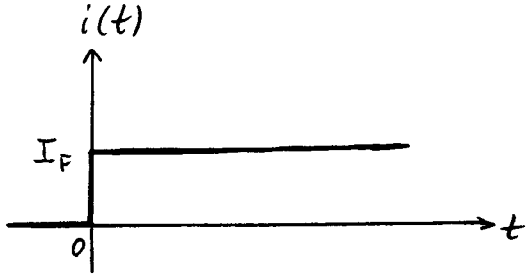
- slope of Δp_n at x_n: $\left.\frac{d\Delta p_n}{dx}\right|_{x=x_n} = -\frac{i}{qAD_p}$ constant and positiv for $0 < t \le t_s$

- storage delay time $t_s \approx \tau_p \ln\left(1 + \frac{I_F}{I_R}\right)$

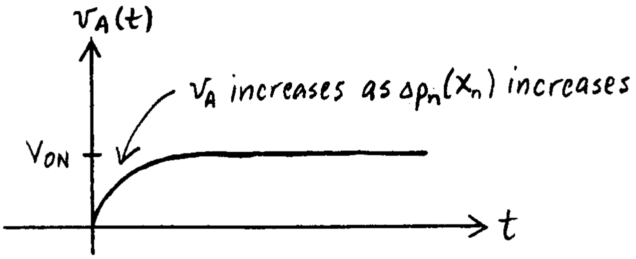
(determined by solving differential equation for Q_p, then approximating $Q_p(t_s) = 0$)

8) DIODE TRANSIENT RESPONSE: TURN-ON

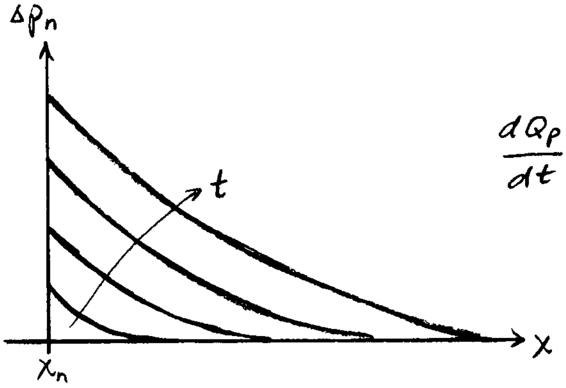




$$V_{ON} = \frac{kT}{q} \ln\left(\frac{I_F}{I_o} + 1\right)$$



Build-up of stored charge inside a p^+n diode:



$$\frac{dQ_p}{dt} = I_F - \frac{Q_p}{\tau_p} \qquad t>0$$

- slope of Δp_N at x_n: $\left.\frac{d\Delta p_N}{dx}\right|_{x=x_n} = -\frac{i}{qAD_p}$ constant, negative for $t>0$

$$v_A(t) = \frac{kT}{q} \ln\left[1 + \frac{I_F}{I_o}\left(1 - e^{-t/\tau_p}\right)\right]$$

(determined by solving D.E. for Q_p, applying boundary condition $v_A(\infty) = V_{ON}$)

$\Rightarrow$ Time required to turn on diode $\simeq \tau_p$

8) SUMMARY OF IMPORTANT CONCEPTS

- Under forward bias, minority carriers are injected into the quasi-neutral regions of the diode.

Current flowing across the junction is comprised of hole and electron current components:

$J_n(-x_p) \sim \frac{D_n n_i^2}{L_n N_a}$ ← parameters associated w/ p-side

$J_p(x_n) \sim \frac{D_p n_i^2}{L_p N_d}$ ← parameters associated w/ n-side

In order for one of these components to dominate, the junction must be asymmetrically doped (i.e.: need a one-sided junction)

p^+n diode ($N_a >> N_d$): $J_p >> J_n$ hence $J \simeq J_p$

n^+p diode ($N_d >> N_a$): $J_n >> J_p$ hence $J \simeq J_n$

- Note: The ideal diode equation stipulates the relationship between $J_n(-x_p)$ and $J_p(x_n)$

$$\frac{J_n}{J_p} = \frac{D_n L_p N_d}{D_p L_n N_a}$$

Therefore, if some holes are forced to flow across the forward-biased junction, then electrons must also be injected across the junction.

- Under <u>reverse bias</u>, minority carriers are <u>collected</u> into the quasi-neutral regions of the diode.

  <u>Minority</u> carriers within a diffusion length of the depletion region will diffuse into the depletion region, and then be swept across the junction by the electric field:

  electrons on p-side ⟶ n-side
  holes on n-side ⟶ p-side

  Current flowing in a reverse-biased diode depends on the rate at which minority carriers are supplied in the quasi-neutral regions (within a diffusion length of the depletion region)