Lecture #12

OUTLINE

– pn Junctions
  • narrow-base diode
  • charge-control model

Reading: Finish Chapter 6
Narrow or Short-Base Diode

• We have the following boundary conditions:

\[ \Delta p_n (x_n) = p_{no} (e^{qV_A/kT} - 1) \quad \Delta p_n (x' = x'_c) \to 0 \]

• With the following coordinate system:

NEW: \[ x'' \quad 0 \quad 0 \quad x' \]

• Then, the solution is of the form:

\[ \Delta p(x) = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p} \]

Applying the boundary conditions, we have:

\[ \Delta p_n (0) = A_1 + A_2 \]
\[ 0 = A_1 e^{x'_c/L_p} + A_2 e^{-x'_c/L_p} \]

• So, we have:

\[ \Delta p_n (x') = p_{no} (e^{qV_A/kT} - 1) \left( \frac{e^{(x'_c-x')/L_p} - e^{(x'_c-x')/L_p}}{e^{x'_c/L_p} - e^{-x'_c/L_p}} \right) , \quad 0 < x' < x'_c \]

• Note: \[ \sinh(\xi) = \frac{e^\xi - e^{-\xi}}{2} \]
• So:
\[
\Delta p_n(x') = p_n(0)(e^{qV_d/kT} - 1) \left( \frac{\sinh\left(\frac{x' - x}{L_p}\right)}{\sinh\left(\frac{x'}{L_p}\right)} \right), \quad 0 < x' < x_c
\]

• If we write:
\[
I = I_0'(e^{qV_d/kT} - 1)
\]

• Then
\[
I_0' = qA \frac{D_p}{L_p} \frac{n_i^2}{N_D} \frac{\cosh\left(\frac{x'}{L_p}\right)}{\sinh\left(\frac{x'}{L_p}\right)}
\]

where \( \cosh(\xi) = \frac{e^\xi + e^{-\xi}}{2} \)

Note: \( \sinh(\xi) \to \xi \) as \( \xi \to 0 \) and \( \cosh(\xi) \to 1 + \xi^2 \) as \( \xi \to 0 \)

• If \( x_c' \ll L_p \):

**Narrow Base Diode: I-V Equation**

Let \( W_N \equiv \text{width of n-type region} \)
\[ W_p \equiv \text{width of p-type region} \]

and \( W'_N = W_N - x_n \ll L_p \)
\[ W'_p = W_p - x_p \ll L_N \]

Then,
\[
J = qn_i^2 \left[ \frac{D_p}{W'_N N_D} + \frac{D_N}{W'_P N_A} \right] \left( e^{qV_A/kT} - 1 \right)
\]

\[
I = qAn_i^2 \left[ \frac{D_p}{W'_N N_D} + \frac{D_N}{W'_P N_A} \right] \left( e^{qV_A/kT} - 1 \right) = I_0 \left( e^{qV_A/kT} - 1 \right)
\]

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**Current Flow in a One-Sided pn Junction**

- Note that the diode current is dominated by the term associated with the more lightly doped side:
  
  \[ I_0 \equiv I_p(x_p) = \begin{cases} 
  qAn_i^2 \left( \frac{D_p}{L_p N_D} \right) & \text{long n-side} \\
  qAn_i^2 \left( \frac{D_p}{W'_N N_D} \right) & \text{short n-side} 
  \end{cases} \]

  \[ I_0 \equiv I_n(-x_n) = \begin{cases} 
  qAn_i^2 \left( \frac{D_N}{L_N N_A} \right) & \text{long p-side} \\
  qAn_i^2 \left( \frac{D_N}{W'_P N_A} \right) & \text{short p-side} 
  \end{cases} \]

  \text{i.e. current flowing across junction is dominated by carriers injected from the more heavily doped side}
Excess Carrier Profiles: Limiting Cases

Long base ($x'_c \to \infty$):

$$\Delta p_n(x') = p_n(0) (e^{qV_d/kT} - 1) \left( \frac{e^{\frac{\phi'_n - x'}{L_p}} - e^{\frac{\phi'_n}{L_p}}}{e^{\frac{\phi'_n}{L_p}} - e^{-\frac{\phi'_n}{L_p}}} \right)$$

$$= p_n(0) (e^{qV_d/kT} - 1) \left( \frac{e^{\frac{\phi'_n}{L_p}} e^{-\frac{\phi'_n}{L_p}} - e^{-\frac{\phi'_n}{L_p}} e^{\frac{\phi'_n}{L_p}}}{e^{\frac{\phi'_n}{L_p}} - e^{-\frac{\phi'_n}{L_p}}} \right)$$

$$\equiv p_n(0) (e^{qV_d/kT} - 1) e^{-\frac{\phi'_n}{L_p}}$$

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2. Short base ($x'_c \to 0$):

$$\Delta p_n(x') = p_n(0) (e^{qV_s/kT} - 1) \left( \frac{\sinh \left( \frac{\phi'_n - x'}{L_p} \right)}{\sinh \left( \frac{\phi'_n}{L_p} \right)} \right)$$

$$= p_n(0) (e^{qV_s/kT} - 1) \left( \frac{\phi'_n - x'}{L_p} \right) = p_n(0) (e^{qV_s/kT} - 1) \left( 1 - \frac{x'}{x'_c} \right)$$

$\Delta p_n$ is a linear function of $x$

$\Rightarrow J_p$ is constant (no recombination)
Minority-Carrier Charge Storage

• When $V_A > 0$, excess minority carriers are stored in the quasi-neutral regions of a pn junction:

$$Q_N = -qA \int_{-x_p}^{\infty} \Delta n_p(x) dx$$

$$Q_p = qA \int_{x_n}^{\infty} \Delta p_n(x) dx$$

$$= -qA \Delta n_p(-x_p)L_N$$

$$= qA \Delta p_n(x_n)L_P$$

Derivation of Charge Control Model

• Consider a forward-biased pn junction. The total excess hole charge in the n quasi-neutral region is:

$$Q_p = qA \int_{x_n}^{\infty} \Delta p_n(x,t) dx$$

• The minority carrier diffusion equation is (without $G_L$):

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

• Since the electric field is very small,

$$J_p = -qD_p \frac{\partial \Delta p_n}{\partial x}$$

• Therefore:

$$\frac{\partial (q\Delta p_n)}{\partial t} = -\frac{\partial J_p}{\partial x} - \frac{q \Delta p_n}{\tau_p}$$
(Long Base Diode)

• Integrating over the n quasi-neutral region:
\[
\frac{d}{dt} \left[ qA \int_{x_n}^x \Delta p_n \, dx \right] = -A \int_{x_n}^x \frac{1}{\tau_p} \left[ qA \int_{x_n}^x \Delta p_n \, dx \right] \]

• Furthermore, in a p+n junction:
\[- A \int_{x_n}^{x_p} dJ_p = -A J_p(\infty) + A J_p(x_n) = A J_p(x_n) \equiv i_{DIFF} \]

• So:
\[ \frac{dQ_p}{dt} = i_{DIFF} - \frac{Q_p}{\tau_p} \]

Charge Control Model

We can calculate pn-junction current in 2 ways:

1. From slopes of \( \Delta n_p(-x_p) \) and \( \Delta p_n(x_n) \)

2. From steady-state charges \( Q_N, Q_P \) stored in each excess-minority-charge distribution:
\[
\frac{dQ_P}{dt} = AJ_p(x_n) - \frac{Q_P}{\tau_p} = 0
\]
\[
\Rightarrow AJ_p(x_n) = I_p(x_n) = \frac{Q_P}{\tau_p}
\]

Similarly, \( I_N(-x_p) = -\frac{Q_N}{\tau_n} \)
Charge Control Model for Narrow Base

- For a narrow-base diode, replace $\tau_p$ and/or $\tau_n$ by the **minority-carrier transit time** $\tau_{tr}$
  - time required for minority carrier to travel across the quasi-neutral region
- For holes on narrow n-side:
  \[ Q_p = qA \int_{x_n}^{w_n} \Delta p_n(x) \, dx = qA \frac{1}{2} \Delta p_n(x_n) W_N' \]
  \[ I_p = A J_p = -q A D_p \frac{d \Delta p_n}{dx} = q A D_p \frac{\Delta p_n(x_n)}{W_N'} \]
  \[ \Rightarrow \tau_{tr} = \frac{Q_p}{I_p} = \frac{(W_N')^2}{2 D_p} \]
- Similarly, for electrons on narrow p-side: $\tau_{tr} = \frac{(W_N')^2}{2 D_n}$

Summary: Narrow (Short) Base Diode

- If the width of the quasi-neutral region (e.g. $x_c'$) is **not** much larger than the minority-carrier diffusion length (e.g. $L_P$), then the solution to the minority-carrier diffusion equation is
  \[ \Delta p_n(x') = \Delta p_n(x_n) \frac{\sinh \left( \frac{x_c' - x}{L_P} \right)}{\sinh \left( \frac{x_c'}{L_P} \right)} \]
- If $x_c' < 0.1 L_P$:
  \[ \Delta p_n(x') \equiv \Delta p_n(x_n) \left( 1 - \frac{x'}{x_c'} \right) \]
• If $L_p \gg x_c'$, negligible recombination occurs
  $\Rightarrow$ hole current is constant throughout n-type region
  \[ J_p = -qD_p \frac{d\Delta p_n}{dx} = qD_p \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right) \left( \frac{1}{x_c'} \right) \]

• Compare this to the hole current contribution in the long-base diode:
  \[ J_p = qD_p \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right) \left( \frac{1}{L_p} \right) \]