Lecture #10

ANNOUNCEMENT
- Quiz #2 next Thursday (2/27) covering
  • carrier action (drift, diffusion, R-G)
  • continuity & minority-carrier diffusion equations
  • metal-semiconductor contacts

OUTLINE
- pn junction electrostatics

Reading: Chapter 5

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pn Junctions

Donors

N-type
P-type

Reverse bias Forward bias

diode symbol

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**Terminology**

Doping Profile:

N_A (diffused) or N_D (original)

Metallurgical junction (N_D - N_A = 0)

**Idealized Junctions**

Actual profile

--- Step junction idealization

--- Linearly graded junction idealization
Energy Band Diagram

Qualitative Electrostatics

Band diagram

Electrostatic potential

Electric field

Charge density
“Game Plan” for Obtaining $\rho(x)$, $\varepsilon(x)$, $V(x)$

- Find the built-in potential $V_{bi}$
- Use the depletion approximation $\rightarrow \rho(x)$ (depletion-layer widths $x_p$, $x_n$ unknown)
- Integrate $\rho(x)$ to find $\varepsilon(x)$
  - boundary conditions $\varepsilon(-x_p)=0$, $\varepsilon(x_n)=0$
- Integrate $\varepsilon(x)$ to obtain $V(x)$
  - boundary conditions $V(-x_p)=0$, $V(x_n)=V_{bi}$
- For $\varepsilon(x)$ to be continuous at $x=0$, $N_A x_p = N_D x_n$
  $\rightarrow$ solve for $x_p$, $x_n$

Built-In Potential $V_{bi}$

\[
q V_{bi} = \Phi_{S-p-side} - \Phi_{S-n-side} = (E_i - E_F)_{p-side} + (E_F - E_i)_{n-side}
\]

For non-degenerately doped material:

\[
(E_i - E_F)_{p-side} = kT \ln \left( \frac{P}{n_i} \right) \quad (E_F - E_i)_{n-side} = kT \ln \left( \frac{n}{n_i} \right)
\]

\[
= kT \ln \left( \frac{N_A}{n_i} \right) \quad = kT \ln \left( \frac{N_D}{n_i} \right)
\]
**V_{bi} for “One-Sided” pn Junctions**

\[ qV_{bi} = (E_i - E_F)_{p-side} + (E_F - E_i)_{n-side} \]

**p+n junction**

**n+p junction**

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**The Depletion Approximation**

On the **p-side**, \( \rho = -qN_A \)

\[
\frac{d\varepsilon}{dx} = -\frac{qN_A}{\varepsilon_s}
\]

\[
\varepsilon(x) = -\frac{qN_A}{\varepsilon_s} x + C_1 = -\frac{qN_A}{\varepsilon_s} (x + x_n)
\]

On the **n-side**, \( \rho = qN_D \)

\[
\varepsilon(x) = -\frac{qN_D}{\varepsilon_s} (x_n - x)
\]
The electric field is continuous at $x = 0$

$$\Rightarrow \quad N_A x_p = N_D x_n$$

Electrostatic Potential in the Depletion Layer

On the $p$-side:

$$V(x) = \frac{qN_A}{2\varepsilon_s} (x + x_p)^2 + D_1$$

(arbitrarily choose the voltage at $x = x_p$ to be 0)

On the $n$-side:

$$V(x) = -\frac{qN_D}{2\varepsilon_s} (x_n - x)^2 + D_2 = V_{bi} - \frac{qN_D}{2\varepsilon_s} (x_n - x)^2$$
• At $x = 0$, expressions for p-side and n-side must be equal:

• We also know that $N_A x_p = N_D x_n$

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**Depletion Layer Width**

• Eliminating $x_p$, we have:

$$x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q} \left( \frac{N_A}{N_D(N_A + N_D)} \right)}$$

• Eliminating $x_n$, we have:

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q} \left( \frac{N_D}{N_A(N_A + N_D)} \right)}$$

• Summing, we have:

$$x_n + x_p = W = \sqrt{\frac{2\varepsilon V_{bi}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)}$$
One-Sided Junctions

If $N_A >> N_D$ as in a p+n junction:

$$W = \frac{2\varepsilon V_n}{q N_D} = x_n$$

$$x_p = x_n N_D / N_A \equiv 0$$

What about a n+p junction?

$$W = \sqrt{2\varepsilon_s V_{bi}/qN}$$

where $\frac{1}{N} = \frac{1}{N_D} + \frac{1}{N_A} = \text{lighter dopant density}$

Peak Electric Field

$$|\int \mathbf{E} \, dx| = \frac{1}{2} |\mathbf{E}(0)| W = V_{bi} - V_A$$

• For a one-sided junction, $W \equiv \sqrt{\frac{2\varepsilon_s}{qN} (V_{bi} - V_A)}$

so $\mathbf{E}(0) = \frac{2(V_{bi} - V_A)}{W} \equiv \sqrt{\frac{2qN(V_{bi} - V_A)}{\varepsilon_s}}$
A p+n junction has $N_A = 10^{20}$ cm$^{-3}$ and $N_D = 10^{17}$ cm$^{-3}$. What is a) its built-in potential, b) $W$, c) $x_n$, and d) $x_p$?

Solution:

a) $V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i} \approx 1$ V

b) $W \approx \sqrt{\frac{2eV_{bi}}{qN_D}} = \left( \frac{2 \times 12 \times 8.85 \times 10^{-14} \times 1}{1.6 \times 10^{-19} \times 10^{17}} \right)^{1/2} = 0.12 \mu m$

c) $x_n \approx W = 0.12 \mu m$

d) $x_p = x_n N_D / N_A = 1.2 \times 10^{-4} \mu m = 1.2 \AA = 0$
Biased PN Junctions

**REQUIREMENT:** $V_A < V_{bi}$, otherwise, we cannot assume low-level injection

Current Flow - Qualitative

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Effect of Bias on Electrostatics

(a) Equilibrium ($V_A = 0$)

(b) Forward bias ($V_A > 0$)

(c) Reverse bias ($V_A < 0$)