Due at 1700, Fri. Oct. 9 in homework box under stairs, first floor Cory.
Note: up to 2 students may turn in a single writeup. Reading Nise 8, 9

1. (20 pts) Root locus (Nise 8.7) moved from PS5, prob 5
Given the unity gain feedback system in Fig. 1, where

\[ G(s) = \frac{K(s+10)(s+20)}{(s+30)(s^2-20s+200)} \]

[4 pts] a) Find and approximately hand sketch the root locus.
[4 pts] b) Find the range of \( K \) which makes the system stable.
Using the second order approximation:
[5 pts] c) Find the value of \( K \) that gives \( \zeta = 0.707 \) for the system’s dominant closed-loop poles.
[5 pts] d) Find the value of \( K \) that will yield a critically damped system.
[2 pts] e) Use MATLAB to plot the step response for c) and d) and compare to approximation estimate.

2. (20 pts) Root locus (Nise 8.8)
The open loop transfer function for a system in unity feedback (Fig. 1) is given by:

\[ G(s) = \frac{s^2+19s-20}{s^2-10s+10c} \]

[4 pts] a) Determine the characteristic equation for the closed loop system.
[16 pts] b) Sketch the root locus with respect to positive values of \( c \), showing direction in which \( c \) increases on the locus.

3. (20 pts) Root Locus (Nise 8.6, 8.8)
Consider the smoothing system shown in Fig. 2. (Assume \( K_1, K_2, K_3 \) are positive constants.)
[7 pts] a. Sketch a root locus where the roots vary as a function of \( K_3 \).
[3 pts] b. Locate the closed-loop zeros.
[10 pts] c. Repeat parts 3a and 3b for a root locus sketched as a function of \( K_2 \).

4. (20 pts) Root locus (Nise 8.6, 8.9)
Consider the unity gain feedback system in Fig. 1 with \( G(s) = \frac{k(s^2+3s+2)}{s^2-1k+5} \). Here \(-\infty < k < \infty\)
[3 pts] b) Find the \( j\omega \) crossing using Routh-Hurwitz.
[3 pts] c) Hand sketch the closed-loop root locus for positive and negative \( k \).
[2 pts] d) Find the range of \( k \) for stability.

5. (20 pts) PD compensation (Nise 9.3)
Consider open loop plant

\[ G(s) = \frac{K}{s(s+10)(s+20)} \]

and unity feedback.
[3 pts] a) find \( K \) such that overshoot is 20%.
[7 pts] b) Design a PD controller such that settling time is reduced by a factor of 4, with the same 20% overshoot.
[6 pts] c) Hand sketch the root locus for the original system and the system with a lead PD compensator, and verify with Matlab.
[2 pts] d) Use Matlab to compare the step response for the closed-loop compensated and uncompensated systems, transient and steady state response.
[2 pts] e) Find the steady state error for a step for both systems.

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Fig. 1. Unity Gain Feedback. Fig. 2. Smoother block diagram.