Professor Fearing EECS128/MEC134 Problem Set 5 v1.1 Fall 2016

Due at 1700, Fri. Oct 2 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise 7,8.

1. (15 pts) Steady state error for non-unity feedback (Nise 7.6)
For the system in Fig. 1, let $G_1(s) = \frac{k}{s^2}$, $G_2(s) = \frac{s+1}{(s+3)}$ and $H(s) = \frac{s+4}{s+2}$, $D(s) = 0$. $E = R - C$.
[3pts] a. What is the system type?
[4pts] b. What is the appropriate static error constant?
[3pts] c. What is the value of the appropriate static eror constant?
[5pts] d. What is the steady state error for a unit step input? For a unit ramp input?

2. (20 pts) Steady state error (Nise 7.8)
[10pts] a) Find steady state error for $r(t)$ a unit step input, using input substitution.
[10pts] b) Find steady state error for $r(t)$ a unit ramp input, using input substitution.
Given system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -21 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \text{ , and } y = [1 \quad 1 \quad 0]\mathbf{x}$$

3. (20 pts) Steady state error (Nise 7)
Consider a PI controller $G_1(s) = k_p + k_i/s$ for plant $G_2(s) = \frac{1}{s+1}$ and $H(s) = 1$, with $r(t) = 0$.
[4pts] a. For a step disturbance $d(t)$, show that the steady state error is zero.
[8pts] b. For a disturbance input $d(t) = \cos(\omega_o t)u(t)$, (e.g. a power line signal) find the steady state error $e(t)$ after all transients have died away. (Note $s = 0$ is not part of region of convergence.) Hint: consider response of system to $e^{j\omega_o t}$, and let $s = j\omega_o$.
[8pts] c. For the same disturbance in b, find a new $G_1(s)$ which will have zero steady state error for a step disturbance, and zero steady state error for $d(t) = \cos(\omega_o t)u(t)$. (Hint: add something else in the controller.)

4. (30 pts) Root locus sketching (Nise 8.6)
For each part below with open loop transfer function $G(s)$ in unity gain feedback (Fig.2):
[5]}i) Apply root locus rules (1-8): specify real axis segments, asymptotes and real axis intercept, break-away and break-in locations on real axis, and angle of departure from complex poles.
[2]}ii) Find $j\omega$ axis intercepts if any.
[1]}iii) Hand sketch root locus.
[1]}iv) Specify range of k for stability.
[1]}v) Verify your root locus using MATLAB.

a) $G(s) = \frac{k(s+5)}{(s+10)(s+2)}$ b) $G(s) = \frac{k(s+5)}{s(s+2)(s+10)^2}$ c) $G(s) = \frac{k(s+5)}{(s+2)(s+7)(s+10)^2}$

`Problem 5 moved to PS6, problem 1.`
5. (15 pts) Root locus (Nise 8.7)
Given the unity gain feedback system in Fig. 2, where

$$G(s) = \frac{K(s+10)(s+20)}{(s+30)(s^2 - 20s + 200)}$$

a) Find and approximately hand sketch the root locus.
b) Find the range of K which makes the system stable.
Using the second order approximation:
c) Find the value of K that gives $\zeta = 0.707$ for the system's dominant closed-loop poles.
d) Find the value of K that will yield a critically damped system.
e) Use MATLAB to plot the step response for c) and d) and compare to approximation estimate.

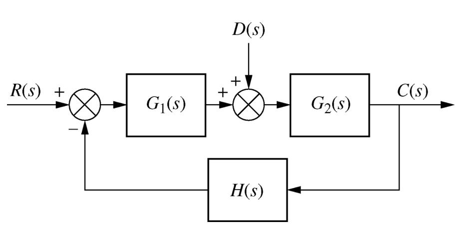

Fig. 1. Control System Block Diagram.

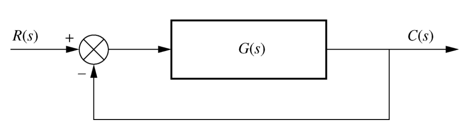

Fig. 2. Unity Gain Feedback.