Professor Fearing EECS128/MEC134 Problem Set 12 v1.0 Fall 2015

**Due at 1700, Fri. Dec. 4 in homework box under stairs, first floor Cory** .
Note: up to 2 students may turn in a single writeup. Reading Nise 13

1. (30 pts) Discrete Time Control (Handout and Matlab)
*For each part, hand in relevant Matlab code as well as plots. Use* `hold on` *to superimpose plots.*
Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -25 & -35 & -11 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t), \quad y = [10\ 1\ 0]\ \mathbf{x}$$

[4pts] a) Find the corresponding discrete time (DT) system $x[n+1] = Gx[n] + Hu[n]$ , $y[n] = Cx[n]$ which can be found using the Matlab function `c2d(sys,T,'zoh')`. Compare eigenvalues for CT $A$ and DT $G$; are they stable?
[4pts] b) With initial condition $x_0 = [1\ \ 0\ \ 0]'$, plot the ZIR using Matlab function `initial()` for the CT system and the DT system (with $T = 0.3$ sec).
[4pts] c) For output feedback control $u = k(r - y)$, sketch the root locus for the equivalent transfer function for the continuous time (CT) system. Determine the closed loop pole locations for the CT system for $k = 20$ and plot the closed-loop step response using Matlab.
[3pts] d) The closed loop DT system has state equation
$x[n + 1] = (G - kHC)x[n] + kHr[n]$ , $y[n] = Cx[n]$
(which can be found using the Matlab `feedback` function). Using Matlab, determine the closed loop pole locations for the DT system for $k = 20$ and sampling period $T = 0.3$ sec and plot the step response on the same axes as part c.
[7pts] e) Use Matlab (iteratively if necessary) to find a sampling period $T$ which gives a closed-loop step response for DT with $k = 20$ that is "reasonably close" to the CT closed-loop step response. Determine closed-loop pole locations, and plot the DT step response on same axes as part d.
[4pts] f) Find the largest $T$ such the DT system with $k = 20$ is marginally stable, and determine closed loop poles.
[4pts] g) Briefly explain why the CT and DT ZIR responses from b) above are reasonably close, but the closed loop responses from c) and d) (with $T = 0.3$ sec) differ significantly. (Hint, consider $G(T) = e^{AT}$.)

2. (15 pts) Discretization (Handout and Matlab)
Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{x}$$

[8 pts] a) Find the corresponding discrete time (DT) system $x[n+1] = Gx[n] + Hu[n]$ , $y[n] = Cx[n]$ by hand calculation, with time step $T = \ln(2)$. Find the eigenvalues of $G$ by hand.
[5 pts] b) Find $x[1], x[2], x[3]$ with $u = 0$ and $x[0] = [0\ 16]^T$ by hand.
[2pts] c) Verify $x[k]$ for b) using Matlab `initial()`.

3. (10 pts) Z-transform (Nise 13.3)
Given $h(kT) = h[k] = (kT)^2 e^{-2kT}$, find $H(z)$ using $H(z) = \Sigma_{k=0}^{\infty} h(kT) z^{-k}$

4. (15 pts) SS to TF (Nise 3.6, 13.3, 13.4, DT handout)

Given the following discrete time (DT) system, with sample period $T = 1$:

$$\mathbf{x}(k+1) = G\mathbf{x}(k) + Hu(k) = \begin{bmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{5}{6} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \tag{1}$$

[5pts] a. Find the transfer function $\frac{X(z)}{U(z)}$.

[2pts] b. Is the system BIBO stable?

[8pts] c. Find $x[n]$ for a unit step input and zero initial conditions using partial fraction expansion.

5. (30 pts) Steady State Error/DT Integrator (Nise 12.8, 13.7, 13.8)

Given the following discrete time (DT) system, with sample period $T = 1$:

$$\mathbf{x}(k+1) = G_1\mathbf{x}(k) + H_1u(k) = \begin{bmatrix} 0 & 1 \\ -\frac{5}{12} & -\frac{4}{3} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k), \quad y = [1 \;\; 0]\ \mathbf{x} \tag{2}$$

[5pts] a) Given error $e(k) = r(k) - y(k)$ where $r(k)$ is a scalar, evaluate the steady state error $\lim_{k\to\infty} e(k)$ for input $r(k)$ a unit step, with state feedback, that is, $u = -K_1\mathbf{x} + r$, where $K_1$ is chosen so that the closed loop poles are at $z_i = 0.5 \pm 0.3j$.

[10pts] b) Add a DT integrator to the plant, with $X_N(k+1) = X_N(k) + e(k)$, where the error $e(k) = r(k) - C\mathbf{x}$. Using a new state vector $\mathbf{x} = [x_1 \; x_2 \; x_N]^T$, write the new state and output equations for DT, equivalent to Nise eq. (12.115ab).

[10pts] c) Find gains such that the 3 closed-loop poles with the DT integrator are at $z_i = 0.5 \pm 0.3j, 0.1$. Evaluate the steady-state error for a step input.

[5pts] d) Plot the step response for both systems in Matlab, (hint `tf(num,den,-1)`) and compare.