EECS C128/ ME C134
Midterm (v. 1.01)
Thurs. Oct. 22, 2015
1540 - 1700 pm

SOLUTIONS

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SID:

- Closed book. One page formula sheet. No calculators.

- There are 5 problems worth 100 points total.

Problem	Points	Score
1	18	
2	19	
3	23	
4	23	
5	17	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$

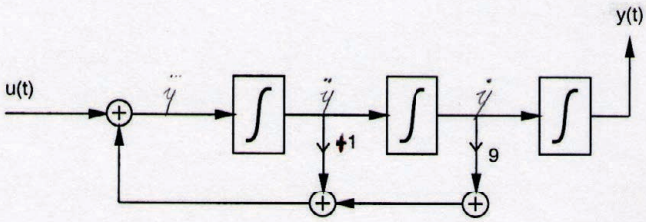
$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

ETHAN SCHALER - SOLUTIONS

Problem 1 (18 pts)

Each part is independent.

[3 pts] a) Consider a single-input single-output system with input u and output y shown in the block diagram below. Assuming zero initial conditions, find the **differential equation** relating $u(t)$ and $y(t)$.



$\frac{d^3y(t)}{dt^3} = u(t) + \ddot{y}(t) + 9\dot{y}$ } 3x 1 pt.

[8 pts] b) A system with input u and state x is described by the differential equation

$$\dot{x} = \frac{1}{(x-4)^2} + 10(u-x)^3$$

Linearize the system about $x = 2$, $u = 2$ and express in the form $\dot{\delta x} = A\delta x + B\delta u$.

$A =$ 1/4

$B =$ 0

$\frac{\partial}{\partial x} f(x,u)\Big|_{x_0,u_0} = -2(x-4)^{-3} + 30(u-x)^2 \Big|_{\substack{x=2\\u=2}}$

$= -2(-2)^{-3} + 30(0)^2$

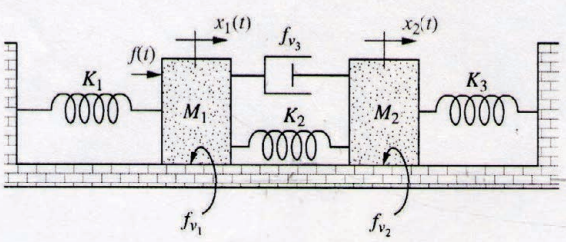
$= 1/4$

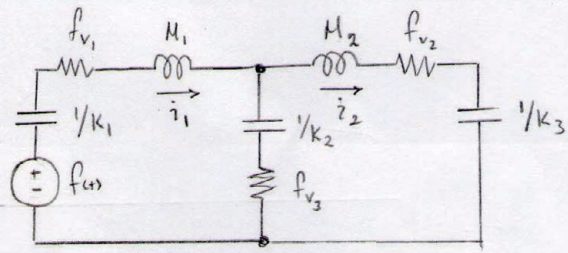
} 4 pts { 3pts. for equation, 1pt for answer

$\frac{\partial}{\partial u} f(x,u) = 0$ } 4pts

-4 pts. - State Space

[7 pts] c) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to force and current to velocity. Let $f_{vi} = R_i$ for $i = 1,2,3$, $\dot{x}_i = i_i$ for $i = 1,2$, $C_i = \frac{1}{K_i}$, for $i = 1,2,3$, $L_1 = M_1$, $L_2 = M_2$.





3 pts - Current Loops & Masses -> Inductors on separate loops.

1 pt - Friction -> Resistors on correct loops.

1pt - Dampers -> Resistors on correct loops.

1pt - Springs -> Caps. on correct loops.

1pt - Force -> Voltage on correct loop

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Problem 2 Steady State Error (19 pts)

[7 pts] a) For the system below, let $H(s) = 1, G_1(s) = \frac{k(s+10)}{s}$, and $G_2(s) = \frac{1}{s+4}$.

For $d(t) = 0$, and $r(t) = tu(t)$ a unit ramp, determine the static error constant, K_v. $K_v = \underline{(10/4)k}$

In unity feedback, so:

$$k_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} sG_1(s)G_2(s)$$
$$= \lim_{s\to 0} \frac{(k)(s+10)\cancel{(s)}}{\cancel{(s)}(s+4)}$$
$$= (10/4)k$$

3pts - k_v equation
3pts - Solving $sG_1(s)G_2(s)$
1pt - Solving $\lim_{s\to 0}$

[7 pts] b) For the system below, let $H(s) = 1, G_1(s) = \frac{k(s+10)}{s}$, and $G_2(s) = \frac{1}{s+4}$.

For $d(t) = tu(t)$, a unit ramp, and $r(t) = 0$, find the steady state expression for $c(t)$ for large t.

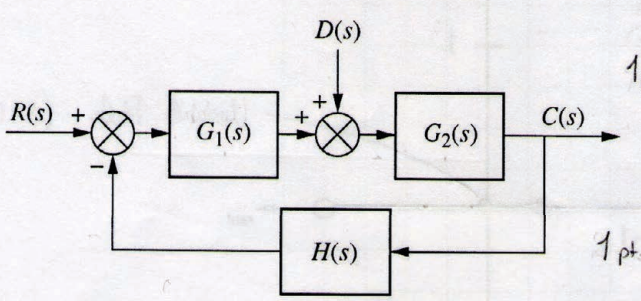
$c(t) = \underline{1/(10k)}$

$$C(\infty) = \lim_{s\to 0} (s)(D(s))\left(\frac{C(s)}{D(s)}\right) = \lim_{s\to 0} (s)\left(\frac{1}{s^2}\right)\left(\frac{\frac{1}{s+4}}{1+\left(\frac{k(s+10)}{(s)(s+4)}\right)}\right)$$
$$= \lim_{s\to 0} \left(\frac{1}{\cancel{s}}\right)\left(\frac{\cancel{(s)}}{(s)(s+4)+k(s+10)}\right)$$
$$= \frac{1}{(k)(10)}$$

-1 pt - Error instead of $c(t)$
-2pts - Missing k
3pts - $C(\infty)$ equation
3pts - Solving $(s)(D(s))\left(\frac{C(s)}{D(s)}\right)$
1 pt - Solving $\lim_{s\to 0}$
OR
4pts - $C(s)$ but no FVT

[5 pts] c) For the system below, let $H(s) = \frac{N_H}{D_H}$, $G_1(s) = \frac{N_{G1}}{D_{G1}}$, and $G_2(s) = \frac{N_{G2}}{D_{G2}}$.

Let $y(t) = r(t) - c(t)$. Find $\frac{Y(s)}{R(s)}$. $\frac{Y(s)}{R(s)} =$ _____



$Y(s) = R(s) - C(s)$

3 pts. $\hookrightarrow \frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1G_2H} = \frac{N_{G_1}N_{G_2}D_H}{D_{G_1}D_{G_2}D_H + N_{G_1}N_{G_2}N_H}$

1pt. $Y(s) = R(s)\left(1 - \frac{N_{G_1}N_{G_2}D_H}{D_{G_1}D_{G_2}D_H + N_{G_1}N_{G_2}N_H}\right)$

1 pt. $$\boxed{\frac{Y(s)}{R(s)} = 1 - \frac{N_{G_1}N_{G_2}D_H}{D_{G_1}D_{G_2}D_H + N_{G_1}N_{G_2}N_H} = \frac{D_{G_1}D_{G_2}D_H + N_{G_1}N_{G_2}N_H - N_{G_1}N_{G_2}D_H}{D_{G_1}D_{G_2}D_H + N_{G_1}N_{G_2}N_H}}$$

-2 pts inverse
-2 pts not simplifying

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Problem 3. Root Locus Plotting (23 pts)

Given open loop transfer function $G(s)$:

$$G(s) = \frac{(s-1)}{(s^2+2s+2)(s+4)}$$

For the root locus $(1 + kG(s) = 0)$ with $k > 0$:

[1 pts] a) Determine the number of branches of the root locus = 3

[2 pts] b) Determine the locus of poles on the real axis [-4, 1]

[2 pts] c) Determine the angles for each asymptote: $\pm\pi/(3-1) = \pm\pi/2$

[3 pts] d) determine the real axis intercept for the asymptotes $s =$ -7/2

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \frac{-4-1-1+\cancel{j}\cancel{j}-1}{2} = -7/2$$

[6 pts] e) Determine the angle of departure for the root locus for the pole at $s = -1 + j =$ 225° or -135°

1pt $\{ \sum \theta_i = \pm\pi \Rightarrow -\theta_s - 90° - \underbrace{\tan^{-1}(1/3)}_{18.4°} + \underbrace{\tan^{-1}(1/-2)}_{180-26.6°} = -\theta_s + 180 - 135 = -\theta_s + 45$

+ 4pts (expansion) $\quad +\theta_s = 180 + 45$

4x1 for poles and zeros

1 pt. $\{ \theta_s = 225°$

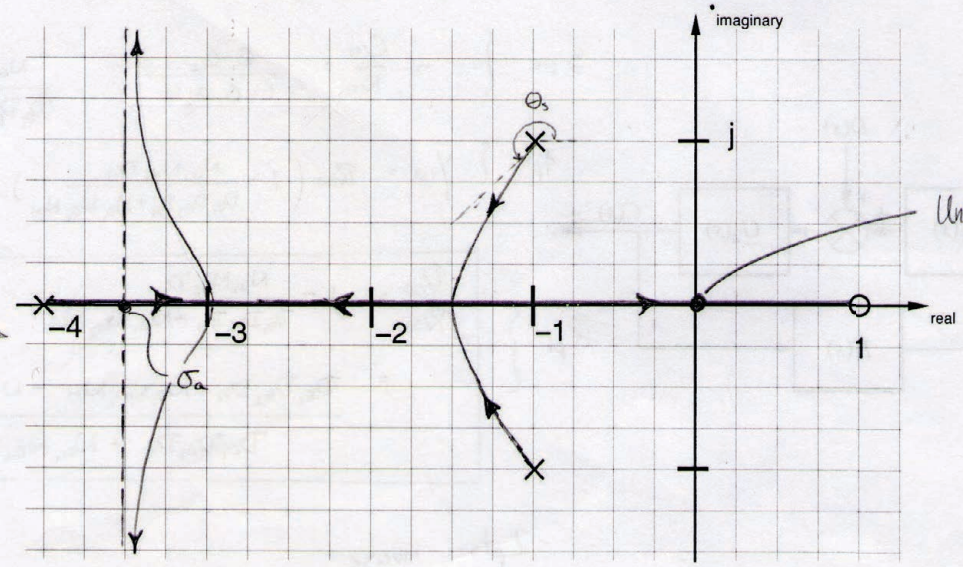
[5 pts] f) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. (Break-In/Break-out points, if any, do not need to be calculated using Rule 6.)

[4 pts] g) Mark the point on the root locus where the system is first unstable. Estimate the minimum value of $k > 0$ for which the closed loop system would be unstable. $k =$ 8

$$k = 1/|G(s)| = 1/\left|\frac{-1}{(2)(4)}\right| = 1/|\tfrac{1}{8}| = 8$$

1pt - Mark on Graph
2pt - Equation to solve
1pt - Solution.

-1 pt per mistake



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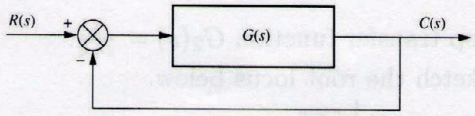
Problem 4. Root Locus Compensation (23 pts)



Given open loop transfer function $G(s)$:

$$G(s) = G_1(s)G_3(s) = G_1(s)\frac{1}{(s+2)^3(s+2+5\sqrt{3})}$$

where $G_3(s)$ is the open-loop plant, and $G_1(s)$ is a lead compensation of the form $G_1(s) = k\frac{s+z_c}{s+p_c}$.

The closed loop system, using unity gain feedback and the lead controller, should have a pair of poles at $p = -2 \pm j\sqrt{3}$.

[4 pts] a. Show that for closed loop poles p, that the angle contribution contribution from $G_3(p)$ is $\approx -280°$.

$$\sum \theta_{G_3} = (-90°)3 - \tan^{-1}(\sqrt{3}/5\sqrt{3})$$
$$= -270 - \tan^{-1}(1/5) \cong -280 \quad (-281.3°) \checkmark$$

2 pts for $\sigma = -2$
2 pts for $\sigma = -2-5\sqrt{3}$

[9 pts] b. Find a lead network pole p_c and zero location z_c such that p is approximately on the root locus, within $\pm$ 10 degrees.

	value	angle	
zero z_c	$2-\sqrt{3}$	135°	2x2 pts
pole p_c	4	-26.6°	2x2 pts
total	—	108.4°	1 pt.

Need $\sum \theta_{G_3} + \sum \theta_{comp} = -180° \ (\pm 360n)$

$\therefore \sum \theta_{comp} = -\theta_{p_c} + \theta_{z_c} = +111.3°$

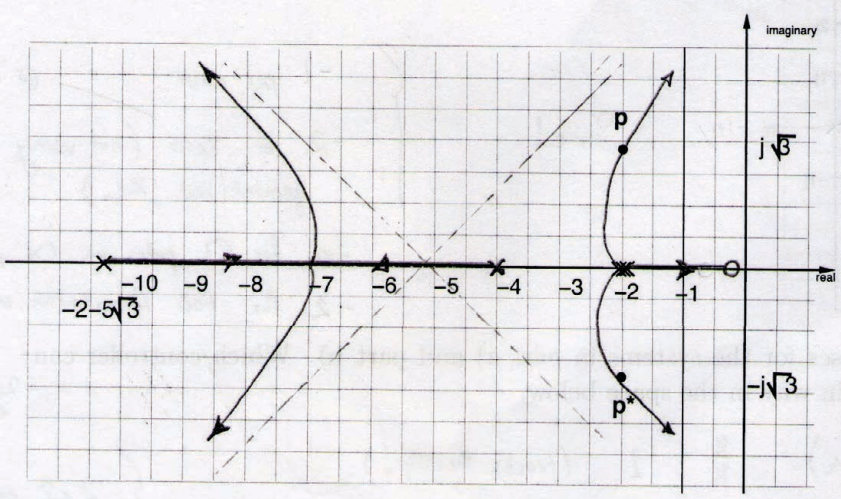
$\hookrightarrow \tan^{-1}(1/-1) = 180 - 45° = 135°$

$\hookrightarrow \tan^{-1}(1/2) = 26.6°$

Place pole @ $p_c = 4$ (pole @ -4) and zero @ $z_c = 2-\sqrt{3}$ (zero @ $-2+\sqrt{3}$) for $\theta_{z_c} = 135$, $\theta_{p_c} = -26.6$ $\Big\}$ $\sum \theta_{comp} = 108.4$

$\angle 10°$ error!

[10 pts] c. For the determined lead network, sketch the root locus, considering real-axis segments, real-axis asymptote intercept, asymptote angles, and locus near p, p^*.



$\theta_a = \pm\pi/4$

$\sigma_a = \frac{(-2)(3) - 4 - 2 - 5\sqrt{3} + (-2+\sqrt{3})}{4}$

$= -\frac{14}{4} - \sqrt{3}$

$\cong -3.5 - 1.73$

$\cong -5.23$

2 pts for σ_a
2 pts for θ_a $\Big\}$ and drawing

1 pt for real-axis segments.

3 pts for locus break-out / assymptote approaches (-1 if not through p) (-1 if bad direction from $(s+2)^3$

2 pts for arrows.

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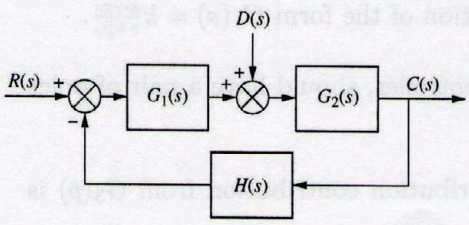
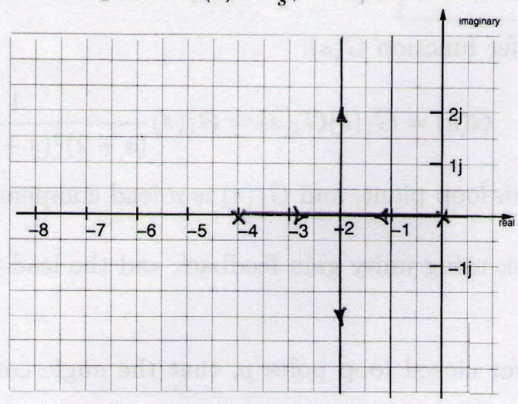
Problem 5. Non-unity gain Compensation (17 pts)

[2 pts] a. Let $D(s) = 0$ (no disturbance). Given plant with open loop transfer function $G_2(s) = \frac{1}{s+4}$. With $H(s) = 1$ and compensator in the feedforward path $G_1(s) = \frac{k}{s}$, sketch the root locus below.

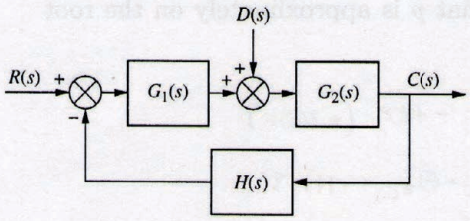
$G_1(s)G_2(s) = \frac{k}{(s)(s+4)}$





-1 per error

Let $D(s) = 0$, $G_1(s) = k$, $G_2(s) = \frac{1}{s+4}$, and controller in the feedback path $H(s) = \frac{1}{s}$.



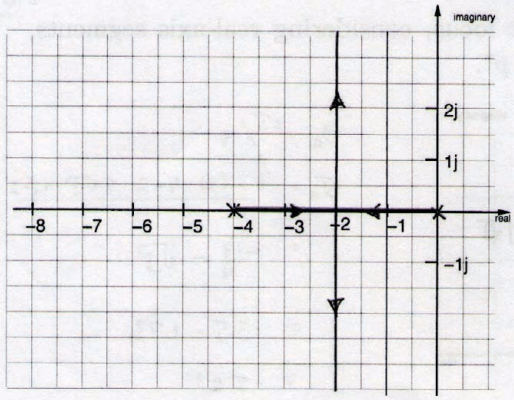
[4 pts] b. Determine the closed loop transfer function for this system $T(s)$:

$$T(s) = \frac{C(s)}{R(s)} = \frac{k(s)}{(s)(s+4) + k}$$

$$T_{cl} = \frac{\left(\frac{k}{s+4}\right)}{1 + (k)\left(\frac{1}{(s)(s+4)}\right)}$$

-1 pts. if missing one term of TF

[6 pts] c. Sketch the root locus for $T(s)$ for $0 \le k < \infty$.



Same!

- -1 per error
- -2 for zero (not using generalized RL) — @ 0
- -2 for CL poles vs. OL poles.
- -2 for bad formulation of general RL

[5 pts] d. Compare the step responses for the systems in part a) and part b). Which controller can better track a step input? Briefly explain why in the space below.

A) $c(\infty) = \lim_{s \to 0} (s)\left(\frac{1}{s}\right)\left(\frac{k}{(s)(s+4)+k}\right) = \frac{k}{k} = 1$ (tracks the step!)

B) $c(\infty) = \lim_{s \to 0} (s)\left(\frac{1}{s}\right)\left(\frac{k(s)}{(s)(s+4)+k}\right) = \frac{0}{k} = 0$ (does not track the step!)

2x2 pts. (-4pts. if no discussion of e_{ss})

-2pts. if test e_{ss} but error

∴ (A) does a better job of tracking step responses, despite identical RL plots. 1 pt.