Name: Ethan Schacer
SID:_________


- There are 5 problems worth 100 points total.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<td>1</td>
<td>18</td>
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<td>2</td>
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<td>5</td>
<td>17</td>
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<td>TOTAL</td>
<td>100</td>
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</table>

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

$$\tan^{-1} \frac{1}{10} = 5.7^\circ$$  $$\tan^{-1} \frac{1}{3} = 11.3^\circ$$

$$\tan^{-1} \frac{4}{1} = 14^\circ$$  $$\tan^{-1} \frac{1}{3} = 18.4^\circ$$

$$\tan^{-1} \frac{1}{2} = 26.6^\circ$$  $$\tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$$

$$\tan^{-1} 1 = 45^\circ$$  $$\tan^{-1} \sqrt{3} = 60^\circ$$

$$\sin 30^\circ = \frac{1}{2}$$  $$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

<table>
<thead>
<tr>
<th>$20 \log_{10} 1$</th>
<th>$20 \log_{10} 2$ = $6dB$</th>
<th>$\pi \approx 3.14$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20 \log_{10} \sqrt{2}$</td>
<td>$20 \log_{10} \frac{1}{2}$ = $-6dB$</td>
<td>$2\pi \approx 6.28$</td>
</tr>
<tr>
<td>$20 \log_{10} 5 = 20db - 6dB = 14dB$</td>
<td>$20 \log_{10} \sqrt{10} = 10 dB$</td>
<td>$\pi/2 \approx 1.57$</td>
</tr>
<tr>
<td>$1/e \approx 0.37$</td>
<td>$\sqrt{10} \approx 3.164$</td>
<td>$\pi/4 \approx 0.79$</td>
</tr>
<tr>
<td>$1/e^2 \approx 0.14$</td>
<td>$\sqrt{2} \approx 1.41$</td>
<td>$\sqrt{3} \approx 1.73$</td>
</tr>
<tr>
<td>$1/e^3 \approx 0.05$</td>
<td>$1/\sqrt{2} \approx 0.71$</td>
<td>$1/\sqrt{3} \approx 0.58$</td>
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</tbody>
</table>
Problem 1 (18 pts)

Each part is independent.

[3 pts] a) Consider a single-input single-output system with input \( u \) and output \( y \) shown in the block diagram below. Assuming zero initial conditions, find the differential equation relating \( u(t) \) and \( y(t) \).

\[
\frac{d^2 y(t)}{dt^2} = u(t) + \ddot{y}(t) + \dddot{y}(t)
\]

\[ \big\{ 3 \times 1 \text{ pt.} \big\]  

[8 pts] b) A system with input \( u \) and state \( x \) is described by the differential equation

\[
\dot{x} = \frac{1}{(x - 4)^2} + 10(u - x)^3
\]

Linearize the system about \( x = 2, u = 2 \) and express in the form

\[
\dot{x} = Ax + Bu.
\]

\[ A = \frac{1}{4} \]

\[ B = 0 \]

\[ \frac{\partial}{\partial x} f(x,u) \left| _{x=u=0} = \begin{bmatrix} -2(x-y)^{-3} + 30(u-x)^2 \end{bmatrix} \right| _{x=2} \]

\[ \frac{\partial}{\partial u} f(x,u) = 0 \]

\[ \begin{align*}
4 \text{ pts} & \quad 3 \text{ pts} \text{ for equation} \\
1 \text{ pt} & \quad \text{for answer} \\
4 \text{ pts} & \quad \text{State Space}
\end{align*} \]

[7 pts] c) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to force and current to velocity. Let \( f_{vi} = R_i \) for \( i = 1, 2, 3 \), \( \dot{x}_i = i_i \) for \( i = 1, 2, 3 \), \( C_i = \frac{1}{K_i} \), for \( i = 1, 2, 3 \), \( L_1 = M_1 \), \( L_2 = M_2 \).

\[ 3 \text{ pts} - \text{Current Loops} & \quad \text{&} \quad \text{Husses} \rightarrow \text{Inductors on separate loops,} \]

\[ 1 \text{ pt} - \text{Fricion} \rightarrow \text{Resistor on correct loops,} \]

\[ 1 \text{ pt} - \text{Dampers} \rightarrow \text{Resistor on correct loops,} \]

\[ 1 \text{ pt} - \text{Springs} \rightarrow \text{Caps. on correct loops,} \]

\[ 1 \text{ pt} - \text{Force} \rightarrow \text{Voltage on correct loop} \]
Problem 2 Steady State Error (19 pts)

[7 pts] a) For the system below, let \( H(s) = 1, G_1(s) = \frac{ke^{+10}}{s}, \) and \( G_2(s) = \frac{1}{s+4}. \)

For \( d(t) = 0, \) and \( r(t) = tu(t) \) a unit ramp, determine the static error constant, \( K_v. \)

\[
K_v = \lim_{s \to 0} sG_2(s) = \frac{(ke^{+10})(s)}{(s)(s+4)} = \frac{(10/4)}{K}
\]

[7 pts] b) For the system below, let \( H(s) = 1, G_1(s) = \frac{ke^{+10}}{s}, \) and \( G_2(s) = \frac{1}{s+4}. \)

For \( d(t) = tu(t), \) a unit ramp, and \( r(t) = 0, \) find the steady state expression for \( c(t) \) for large \( t. \)

\[
c(t) = \frac{1}{(10 \cdot K)}
\]

[5 pts] c) For the system below, let \( H(s) = \frac{N_c}{D_H}, G_1(s) = \frac{N_{c1}}{D_{c1}}, \) and \( G_2(s) = \frac{N_{c2}}{D_{c2}}. \)

Let \( y(t) = r(t) - c(t). \) Find \( \frac{Y(s)}{R(s)}. \)

\[
\frac{Y(s)}{R(s)} = \frac{N_{c2} - N_{c1}}{1 + G_{c1} \cdot G_{c2}}
\]
Problem 3. Root Locus Plotting (23 pts)

Given open loop transfer function \( G(s) \):

\[
G(s) = \frac{s - 1}{(s^2 + 2s + 2)(s + 4)}
\]

For the root locus \( 1 + kG(s) = 0 \) with \( k > 0 \):

[1 pts] a) Determine the number of branches of the root locus = 3

[2 pts] b) Determine the locus of poles on the real axis \([-4, 1]\)

[2 pts] c) Determine the angles for each asymptote: \( \frac{\pi}{2} \) or \( \frac{\pi}{2} \)

[3 pts] d) determine the real axis intercept for the asymptotes \( s = \frac{-7}{2} \)

\[
\sigma_a = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{n - m} = \frac{-4 - 1 + 1 - 1 - 1}{2} = \frac{-7}{2}
\]

[6 pts] e) Determine the angle of departure for the root locus for the pole at \( s = -1 + j = \frac{225}{180}^\circ \) or \( -135^\circ \)

\[
\sum \Theta_i = \pm \pi \Rightarrow \Theta_3 = 90^\circ - \tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{2}) = -\Theta_4 + 180 - 135^\circ = -\Theta_5 + 45^\circ
\]

[5 pts] f) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. (Break In/Break out points, if any, do not need to be calculated using Rule 6.)

[4 pts] g) Mark the point on the root locus where the system is first unstable. Estimate the minimum value of \( k > 0 \) for which the closed loop system would be unstable. \( k = \frac{8}{9} \)
Problem 4. Root Locus Compensation (23 pts)

Given open loop transfer function $G(s)$:

$$G(s) = G_1(s)G_3(s) = G_1(s) \frac{1}{(s+2)^3(s+2+5\sqrt{3})}$$

where $G_3(s)$ is the open-loop plant, and $G_1(s)$ is a lead compensation of the form $G_1(s) = \frac{k(s+a)}{s+b}$.

The closed loop system, using unity gain feedback and the lead controller, should have a pair of poles at $p = -2 \pm j\sqrt{3}$.

[4 pts] a. Show that for closed loop poles $p$, that the angle contribution contribution from $G_3(p)$ is $\approx -280^\circ$.

$$\sum \Theta_{G_3} = (-90^\circ) = - \tan^{-1}\left(\frac{\sqrt{3}}{2\sqrt{3}}\right)$$

$$-270^\circ - \tan^{-1}\left(\frac{1}{5}\right) \approx -280^\circ \left(-281.2^\circ\right)$$

\[2 \text{ ph for } 5 = -2\]
\[2 \text{ ph for } 5 = -2.5\sqrt{3}\]

[9 pts] b. Find a lead network pole $p_c$ and zero location $z_c$ such that $p$ is approximately on the root locus, within $\pm 10$ degrees.

<table>
<thead>
<tr>
<th>value</th>
<th>angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero $z_c$</td>
<td>$2 - \sqrt{3}$</td>
</tr>
<tr>
<td>pole $p_c$</td>
<td>4</td>
</tr>
<tr>
<td>total</td>
<td></td>
</tr>
</tbody>
</table>

Need $\sum \Theta_{G_3} - \sum \Theta_{\text{comp}} = -180^\circ$ ($\pm 360^\circ$)

$$\sum \Theta_{\text{comp}} = - \Theta_{p_c} + \Theta_{z_c} = +111.3^\circ$$

$$\tan^{-1}\left(\frac{1}{-1}\right) = 180^\circ - 45^\circ \div 135^\circ$$

$$\tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$$

Place pole @ $p_c = 4$ (pole @ -4) and zero @ $z_c = 2 - \sqrt{3}$

$$\Theta_{z_c} = \frac{135^\circ}{2}, \Theta_{p_c} = 26.6^\circ$$

\[2 \text{ ph for } \Theta_{z_c}, \text{ drawing}\]
\[2 \text{ ph for } \Theta_{p_c}\]
\[1 \text{ ph for real-axis segments}\]
\[3 \text{ ph for locus break-out/aspymptote } \Theta_a \text{ approach } \left(\Theta \text{ not through } p\right)\]
\[2 \text{ ph for arrows}\]

[10 pts] c. For the determined lead network, sketch the root locus, considering real-axis segments, real-axis asymptote intercept, asymptote angles, and locus near $p, p^\star$.

\[
\Theta_a = \frac{\pi}{4}
\]

\[
S_a = \frac{-2(3) - 2 - 5\sqrt{3}}{4} + (2 + \sqrt{3})
\]

\[= -14 - \sqrt{3}\]

\[= -3.5 - 1.73\]

\[= 5.23\]

\[2 \text{ ph for } \Theta_{a}, \text{ drawing}\]
\[2 \text{ ph for } \Theta_a\]
\[1 \text{ ph for real-axis segments}\]
\[3 \text{ ph for locus break-out/asymptotic approaches } \left(\Theta \text{ not through } p\right)\]
\[2 \text{ ph for arrows}\]
Problem 5. Non-unity gain Compensation (17 pts)

[2 pts] a. Let \( D(s) = 0 \) (no disturbance). Given plant with open loop transfer function \( G_2(s) = \frac{1}{s+4} \). With \( H(s) = 1 \) and compensator in the feedforward path \( G_1(s) = \frac{k}{s} \), sketch the root locus below.

\[ G_1(s) + \frac{k}{s} \]

Let \( D(s) = 0 \), \( G_1(s) = k \), \( G_2(s) = \frac{1}{s+4} \), and controller in the feedback path \( H(s) = \frac{1}{s} \).

[4 pts] b. Determine the closed loop transfer function for this system \( T(s) \):

\[ T(s) = \frac{C(s)}{R(s)} = \frac{k(s)}{(s)(s+4) + k} \]

\( T_{in} > \frac{\left(\frac{k}{s+4}\right)}{1 + \left(\frac{k}{s+4}\right)} \]

-1 pt. if using one term of \( TF \)

[6 pts] c. Sketch the root locus for \( T(s) \) for \( 0 \leq k < \infty \).

[5 pts] d. Compare the step responses for the systems in part a) and part b). Which controller can better track a step input? Briefly explain why in the space below.

A) \( C(\infty) \cdot \lim_{s \to 0} \frac{4}{(s)(4s+1) + k} = \frac{4}{k} = 1 \) (tracks the step!)

B) \( C(\infty) \cdot \lim_{s \to 0} \frac{4}{(s)(4s+1) + k} > \frac{0}{k} = 0 \) (does not track the step!)

So (A) does a better job of tracking step responses, despite identical RL plots. 1 pt.