Lecture abstract

Topics covered in this presentation
- FR compensator design advantages
- Lag, lead, & lag-lead compensators

Chapter outline

11 Design via frequency response
- 11.1 Introduction
- 11.2 Transient response via gain adjustment
- 11.3 Lag compensation
- 11.4 Lead compensation
- 11.5 Lag-lead compensation

FR design review, [1, p. 627]

Stability
- Nyquist criterion → stability
  - CL stable if OL stable & OL magnitude FR has a gain less than 0 dB at the frequency where the phase FR is 180°

TR
- ↓ %OS ∝ φ_M
- ↑ speed of response ∝ ↑ bandwidth

Steady-state error
- ↓ steady-state error ∝ ↑ low-frequency magnitude responses
11 Design via frequency response

11.1 Introduction
11.2 Transient response via gain adjustment
11.3 Lag compensation
11.4 Lead compensation
11.5 Lag-lead compensation

**Concept**

- \( \zeta (\% OS) \) relate to \( \Phi_M \)

\( G(s) = \frac{\omega_n^2}{s(s + 2\zeta \omega_n)} \)

CL TF

\( T(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \)

\( \zeta - \Phi_M \) relation

\( \Phi_M = \tan^{-1} \left( \frac{2\zeta}{\sqrt{1 - 4\zeta^2}} \right) \)

*Figure: Bode plots showing gain adjustment for a desired \( \Phi_M \)*

**Procedure**

1. Draw the Bode magnitude & phase plots for a convenient value of gain
2. Determine the required \( \Phi_M \) from the \( \% OS \)

*Figure: Bode plots showing gain adjustment for a desired \( \Phi_M \)*

**Example (TR design via gain adjustment)**

*Problem:* For the position control system, find the value of the preamplifier gain, \( K \), to yield \( \% OS = 9.5\% \) in the TR for a step input

*Solution:* On the board

*Figure: System*
11 Design via FR
11.3 Lag compensation

Visualizing lag compensation, [1, p. 630]

Concept
- Improve the static error constant by \[ \uparrow \text{only the low-frequency gain} \]
  without any resulting instability
- \[ \uparrow \Phi_M \] of the system to yield the desired TR

Procedure
1. Set the gain, \( K \), to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain

2. Find the frequency where \( \Phi_M \) is \[ 5^\circ \text{ to } 12^\circ \] greater than the \( \Phi_M \) that yields the desired TR. This compensates for the fact that the phase of the lag compensator may still contribute anywhere from \[ -5^\circ \text{ to } -12^\circ \] of phase at \( \omega \Phi_M \)

3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through \( 0 \text{ dB} \) at the frequency found in Step 2

Figure: Visualizing lag compensation
**Visualizing lag compensation, [1, p. 630]**

**Procedure**

3.3 Select the low-frequency asymptote to be at 0 dB.

3.4 Connect the compensator’s high & low-frequency asymptotes with a –20 dB/decade line to locate the lower break frequency.

**Result**

- Lag compensator TF
  \[ G_C(s) = \frac{s + \frac{1}{\alpha T}}{s + \frac{1}{T}} \]
  - \( \alpha > 1 \)

**Example, [1, p. 632]**

**Example (Lag compensation design)**

- **Problem:** Use Bode diagrams to design a lag compensator to yield a tenfold improvement in steady-state error over the gain-compensated system while keeping \( \%OS = 9.5\% \).
- **Solution:** On the board

**Concept**

- Change the phase diagram
- \( \uparrow \text{gain crossover} \iff \uparrow \text{bandwidth} \)
- \( \uparrow \Phi_M \iff \%OS \)
- \( \uparrow \Phi_M \iff \downarrow T_p \)
- Implement a steady-state error requirement \( \rightarrow \) design a TR
11 Design via FR
11.4 Lead compensation

Visualizing lead compensation, [1, p. 635]

**Concept**
- Lead compensator TF
  \[ G_C(s) = \frac{1}{\beta s + 1} \]
  - Frequency at maximum phase shift angle
    \[ \omega_{\text{max}} = \frac{1}{2\sqrt{\beta}} \]

**Procedure**
1. Find the CL bandwidth required to meet a \( T_s, T_p, \text{or} T_r \) requirement
   \[ \omega_{BW} = \omega_n \sqrt{1 - 2\zeta^2} + \sqrt{4\zeta^4 - 4\zeta^2 + 2} \]
   \[ \omega_n = \frac{4}{T_s \zeta} \quad \text{and} \quad \omega_n = \frac{\pi}{T_p\sqrt{1 - \zeta^2}} \]
2. Since the lead compensator has negligible effect at low frequencies, set the gain, \( K_v \), of the uncompensated system to the value that satisfies the steady-state error requirement
3. Plot the Bode diagrams for this value of gain and determine the uncompensated system’s \( \Phi_M \)
4. Determine the new \( \Phi_M \) by finding where the uncompensated system’s magnitude curve is the negative of the lead compensator’s magnitude at the peak of the compensator’s phase curve
5. Design the lead compensator’s break frequencies to find \( T \) and the break frequencies
   \[ G_C(s) = \frac{1}{\beta s + 1} \]
   \[ \omega_{\text{max}} = \frac{1}{2\sqrt{\beta}} \]

**Example (Lead compensation design)**
- Problem: Design a lead compensator to yield \( \%OS = 20\% \), \( K_V = 40 \), \& \( T_p = 0.1 \) second
- Solution: On the board

**Figure:** System
11 Design via frequency response

11.5 Lag-lead compensation

Concept

- What we are not doing: separate lag & lead compensators
  1. Design a lag compensator to lower the high-frequency gain, stabilize the system, & improve the steady-state error
  2. Design a lead compensator to meet the phase-margin requirements
- What we are doing: passive lag-lead network
  - Eliminates the buffer amplifier that separates the lag network from the lead network

Visualizing lag-lead compensation, [1, p. 641]

Procedure

1. Using a 2nd-order approximation, find the CL bandwidth required to meet $T_s$, $T_p$, or $T_r$ [1, p. 582]
2. Set the gain, $K$, to the value required by the steady-state error specification
3. Plot the Bode diagrams for this value of gain
4. Using a 2nd-order approximation, calculate the $M$ to meet the $\%OS$ requirement [1, p. 590]
5. Select a new $\omega_M$ near $\omega_{BW}$
6. At the new $\omega_M$, determine the additional amount of phase lead required to meet the $\Phi_M$ requirement. Add a small contribution that will be required after the addition of the lag compensator.

Visualizing lag-lead compensation, [1, p. 643]

Procedure

7. Design the lag compensator by selecting the higher break frequency one decade below the new $\omega_M$. The design of the lag compensator is not critical, and any design for the proper $\Phi_M$ will be relegated to the lead compensator. The lag compensator simply provides stabilization of the system with the gain required for the steady-state error specification. Find the value of $\gamma$ from the lead compensator’s requirements. Using the phase required from the lead compensator, the phase response curve can be used to find the value of $\gamma = \beta^{-1}$. This value, along with the previously found lag’s upper break frequency, allows us to find the lag’s lower break frequency.

8. Design the lead compensator. Using the value of $\gamma$ from the lag compensator design and the value assumed for the new $\omega_M$, find the lower- and upper-break frequency for the lead compensator.

9. Check the bandwidth to be sure the speed requirement has been met
10. Redesign if $\Phi_M$ or TR specifications are not met, as shown by analysis or simulation
Example (Lag-lead compensation design)

- **Problem:** Design a passive lag-lead compensator using Bode diagrams to yield $\%OS = 13.25\%$, $T_p = 2$ seconds, & $K_v = 12$

  $$G(s) = \frac{K}{s(s+1)(s+4)}$$

- **Solution:** On the board

Bibliography