Professor Fearing EECS C128/ME C134 Problem Set 7 Fall 2010

Due at 10:00 am, Thurs. Nov. 4 in homework box in 240 Cory Hall . Note: up to 2 students may turn in a single writeup.

Reading FPE Ch. 6.7.

Please note how long each problem required to complete.

1. (35 pts) Lag compensation.

For open loop plant

$$G(s) = \frac{1}{s(s+5)(s+8)}$$

a) Find compensator gain $D(s) = k$ such that the unity gain feedback system has steady state error $\lim_{t\to\infty}\ e(t) < 0.2$ for a ramp input $r(t) = tu(t)$.

b) With the gain k above, use a MATLAB Bode plot to determine phase margin.

c) Use MATLAB to find a lag compensator $D(s) = \frac{k_{lag}}{\alpha}\frac{s+1/T}{s+1/\alpha T}$ with appropriate gain k_{lag} so as to obtain phase margin of 45° and unity gain frequency $\omega_c > 2$ rad/sec, with steady state error from ramp input $<= 0.2$.

d) Using MATLAB, plot the steady state error for a ramp input for the closed-loop gain compensated and lag compensated systems.

2. (10 pts) State Space

For the following transfer function, give state space description (with state variable $\mathbf{x}$) in controllable canonical form:

$$\frac{Y(s)}{U(s)} = \frac{2s^3 + 4s^2 + 6s + 8}{s^3 + 8s^2 + 13s + 6}$$

3. (10 pts) State space

Convert the following differential equation to state space in controllable canonical form (with state variable $\mathbf{x}$), where u is the input and y is the output.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = u + \frac{du}{dt}$$

4. (20 pts) State space to transfer function

Given

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) \qquad \text{and } \ y = [-\frac{1}{2} \quad \frac{3}{2} \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a) Draw a block diagram for this system with input u and output y.

b) Starting from the block diagram, determine the transfer function $\frac{Y(s)}{U(s)}$.

5. (25 pts) State space solutions

Given the following

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [3\ 2]\ \mathbf{x} \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solve for the state transition matrix e^{At}, $\mathbf{x}(t)$, and $y(t)$, where $u(t)$ is the unit step.