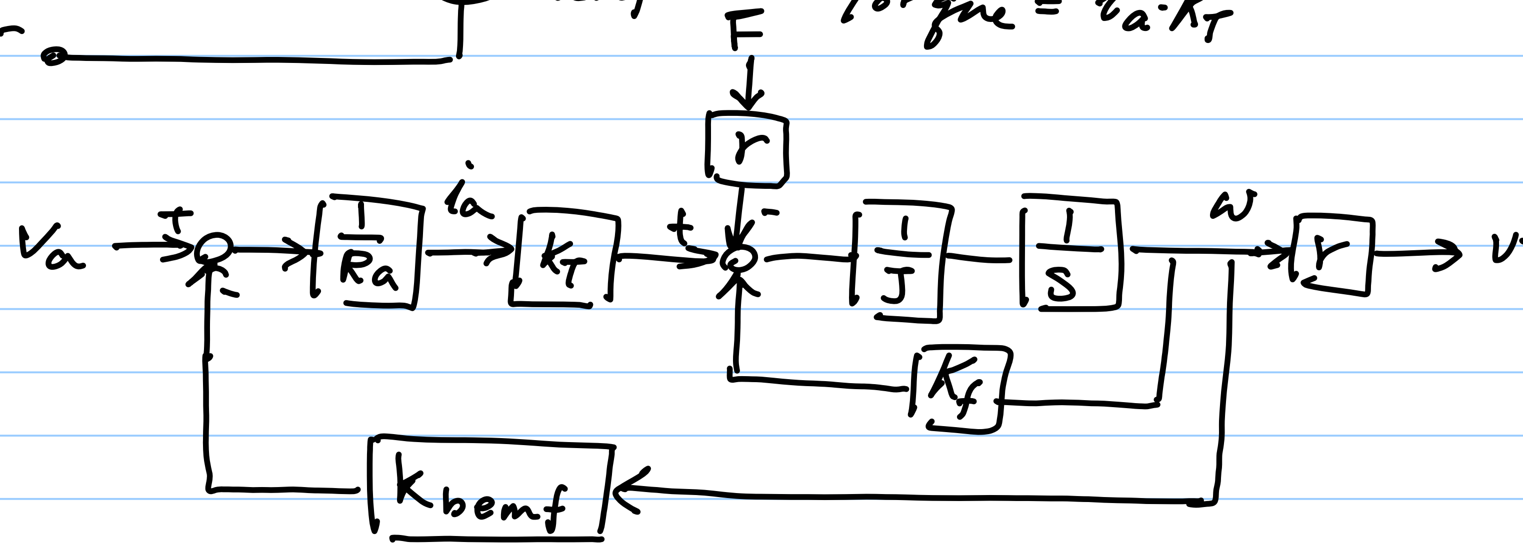
Note Title 9/26/2007

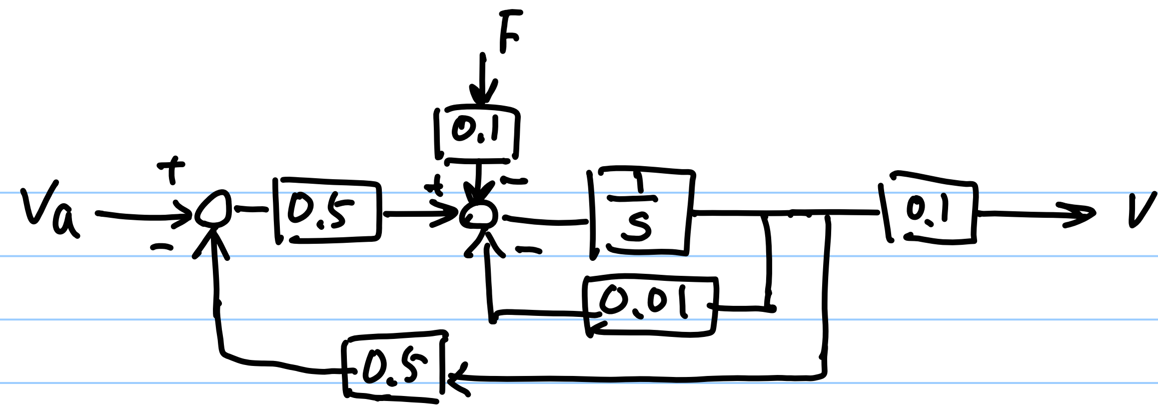
Fall 07 Midterm #1 Solution

①

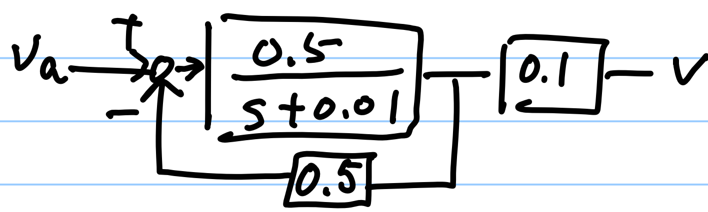
$$i_a = \frac{1}{R_a}\left(V_a - K_{bemf} \cdot \omega\right)$$

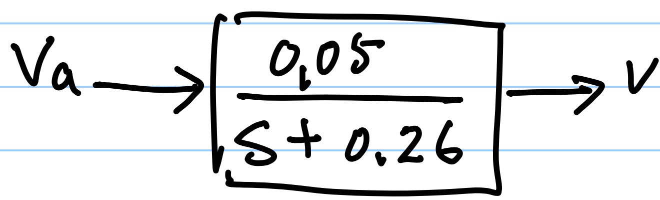
$$Torque = i_a \cdot K_T$$





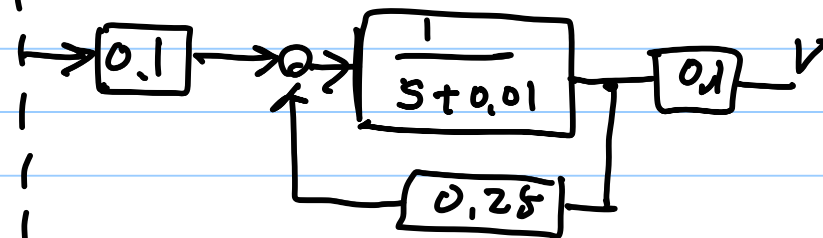
$H(s)$:

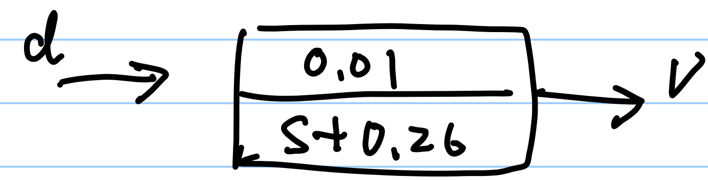




$$\text{DC gain} = \frac{5}{26}$$

$H_d(s)$





$$\text{DC gain} = \frac{1}{26}$$

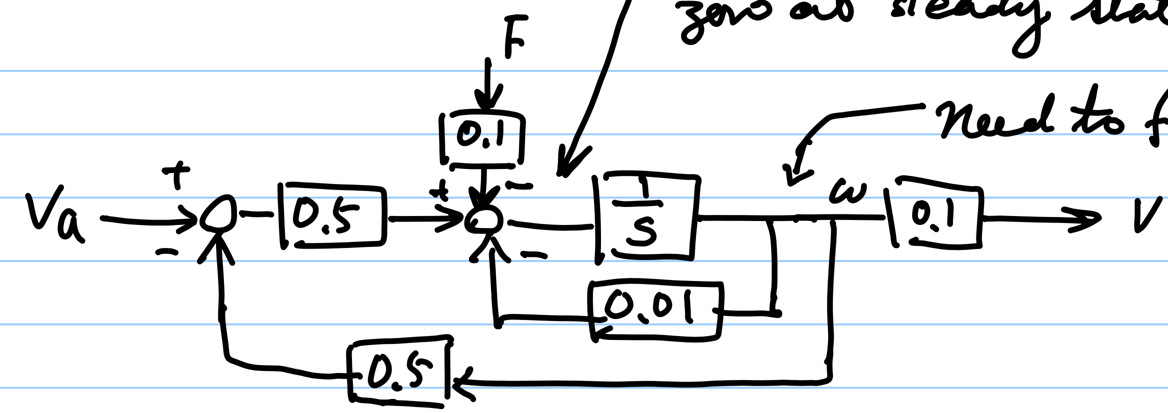
With $V_a = 10$. $F = 10$

$$V = (10)\left(\frac{5}{26}\right) - \frac{1}{26}(10) = \frac{20}{13} \quad (m/s)$$

Alt. method:

Input to a integrator must be zero at steady state.

Need to find ω



$$0 = (V_a)(0.5) - (0.5)(0.5)(\omega) - (0.01)\omega - F(0.1)$$

where $V_a = 10$, $F = 10$

$$0 = 5 - 0.26\omega - 1$$

$$\Rightarrow \quad \omega = \frac{200}{13}\,(r/s) \quad , V = (0.1)\omega = \frac{20}{13}\,(m/s)$$

(2) $mL^2 \ddot{\theta} = -mgL \sin\theta + u$

Let $q_1 = \theta$, $q_2 = \dot{\theta}$

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} q_2 \\ -\frac{g}{L} \sin(q_1) + \frac{1}{mL^2} u \end{pmatrix} , \quad y = L(1 - \cos q_1)$$

(2,b) To make $\theta = 135^0$ an equilibrium point,

$$-\frac{g}{L} \sin(135^\circ) + \frac{1}{mL^2} u_0 = 0 , \quad u_0 = mgL \sin(135^\circ)$$

$u_0 = (9.8)(0.707)$,

Linearized equation :

$$\begin{pmatrix} \dot{\tilde{q}}_1 \\ \dot{\tilde{q}}_2 \end{pmatrix} = \begin{pmatrix} \tilde{q}_2 \\ -9.8 \cos(135^\circ)\tilde{q}_1 + \tilde{u} \end{pmatrix} \qquad \begin{aligned} \tilde{q}_1 &= q_1 - \tfrac{3}{4}\pi \\ \tilde{q}_2 &= q_2 \end{aligned}$$

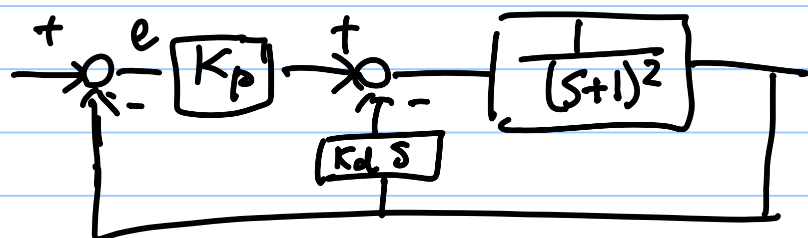
$$u = u_0 - K_1 \tilde{q}_1 - K_2 \tilde{q}_2$$

when $K_2 > 0$, for damping

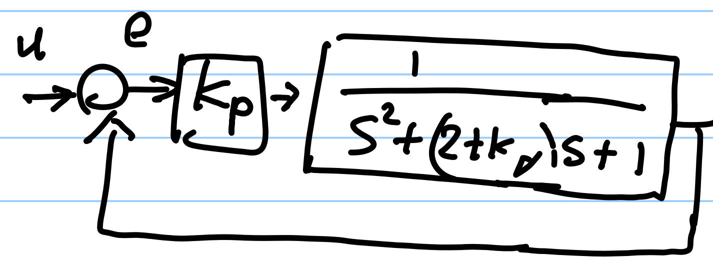
$K_1 > 9.8 \cos(\frac{3}{4}\pi)$, for negative feedback from θ.

$u_0 = (9.8)(0.707)$, for making $\frac{3}{4}\pi$ an eq pt.

(3)



$\Rightarrow$

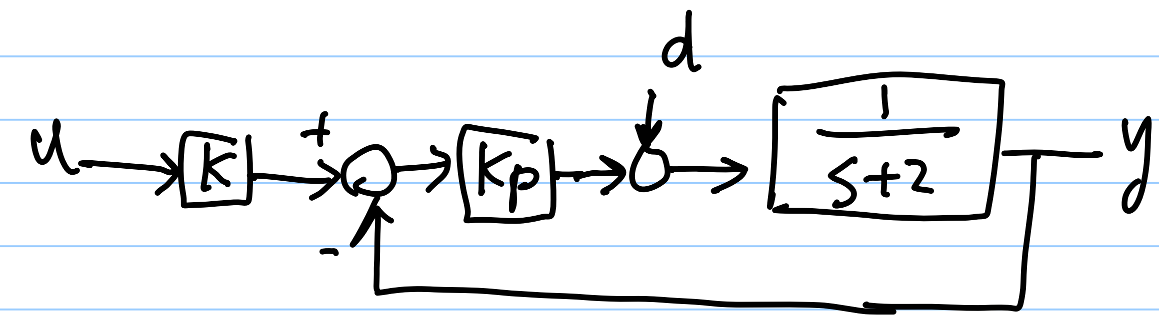


$$\Rightarrow H_e(s) = \frac{s^2+(2+k_v)s+1}{s^2+(2+k_v)s+1+k_p} \qquad H_e(0) = \frac{1}{1+k_p} < 0.02 \Rightarrow k_p = 50$$

For 5% overshoot . $\zeta = 0.7$

$$2 + K_d = 2 \cdot 0.7 \cdot \sqrt{50} \quad \Rightarrow \quad K_d = 7.9$$

(4)



$$H_d = \frac{\frac{1}{s+2}}{1 + \frac{K_p}{s+2}} = \frac{1}{s+2+K_p}$$

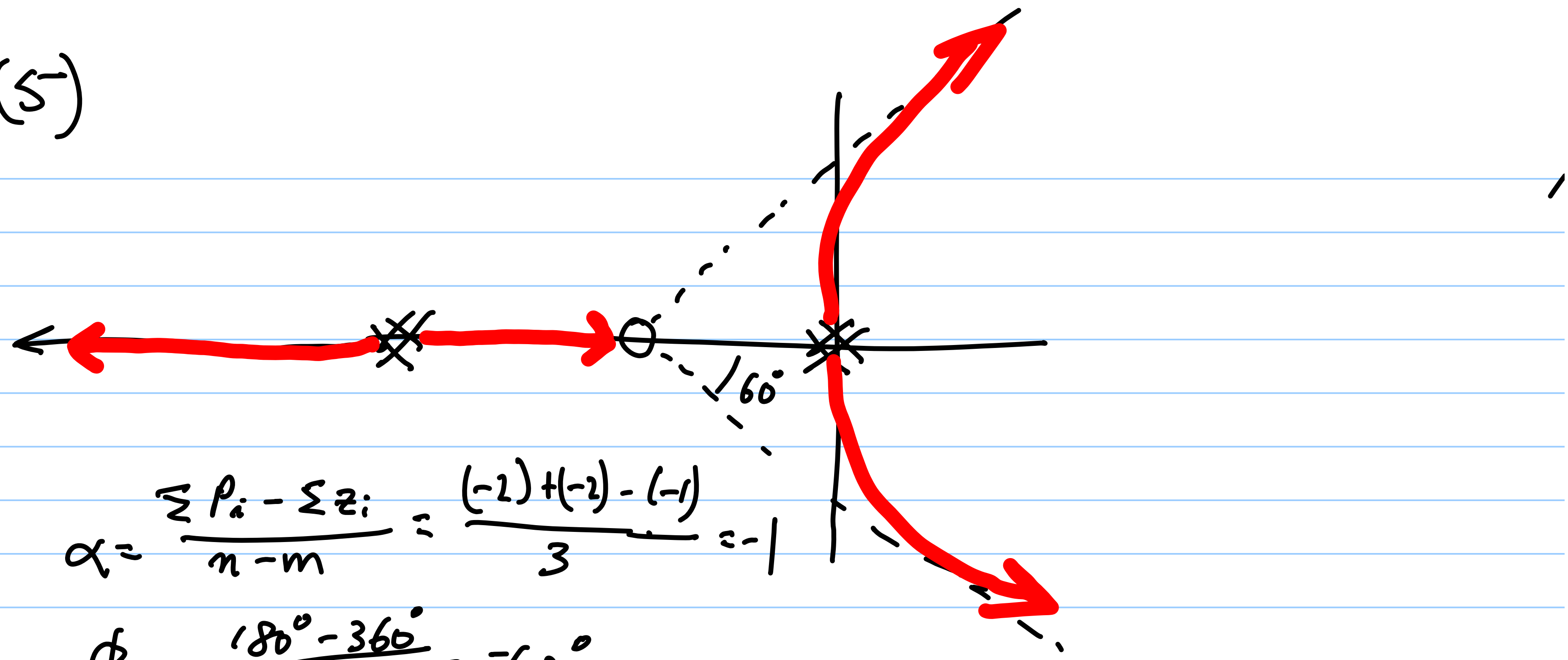
$$H_d(0) = \frac{1}{2+K_p} = 0.05$$

$$\Rightarrow K_p = 18$$

$$H_u(s) = K\left(\frac{\frac{18}{s+2}}{1 + \frac{18}{s+2}}\right) = \frac{K \cdot 18}{s+2+18}$$

$$H_u(0) = \frac{K \cdot 18}{20} = 2 \Rightarrow K = \frac{20}{9}$$

(5)



$$\alpha = \frac{\sum p_i - \sum z_i}{n-m} = \frac{(-2)+(-2)-(-1)}{3} = -1$$

$$\phi_i = \frac{180^\circ - 360^\circ}{3} = -60^\circ$$

(6) ① No zero on the RHP or the imaginary axis.

(RHP poles are ok.)

② $(\sum P_i - \sum z_i) < 0$