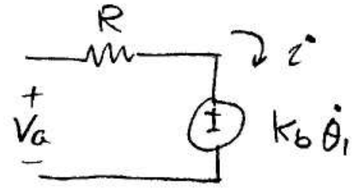
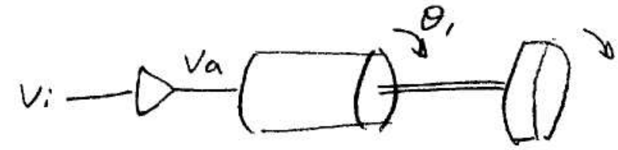
EE128 F05 Midterm Solution

(1)





$$i = \frac{V_a - k_b \dot{\theta}_1}{R} = \frac{k_a V_i - k_b \dot{\theta}_1}{R}$$

Net Torque on the motor rotor:

$$T_M = \frac{k_t (k_a V_i - k_b \dot{\theta})}{R} + k_s (\theta_2 - \theta_1)$$

Net Torque on the load:

$$T_L = k_s (\theta_1 - \theta_2) - B \dot{\theta}_2$$

Motor rotor dynamics: (Newton's law)

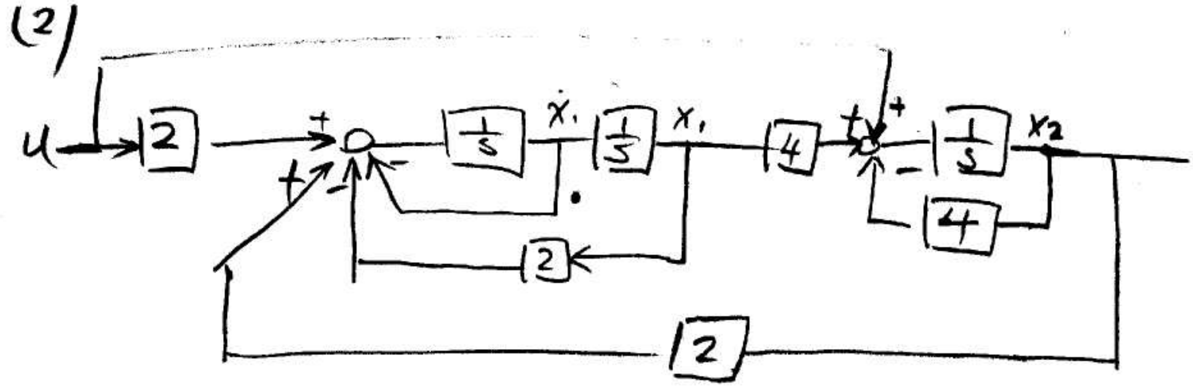
$$J_1 \theta_1'' = \frac{k_t (k_a V_1 - k_b \dot{\theta})}{R} + k_s (\theta_2 - \theta_1)$$

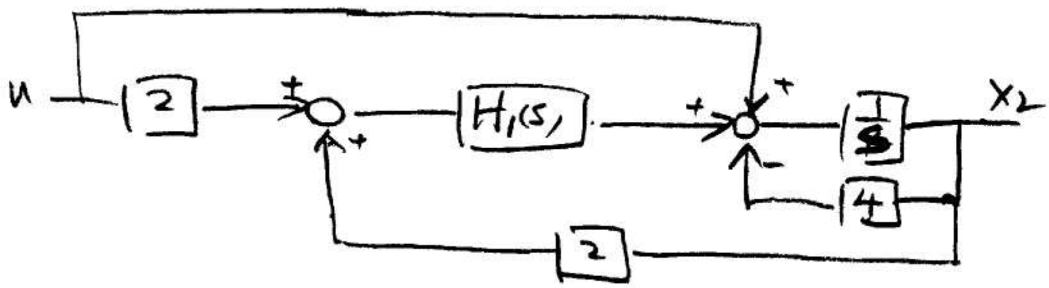
Load dynamics: $J_2 \theta_2'' = k_s (\theta_1 - \theta_2) - B \dot{\theta}_2$

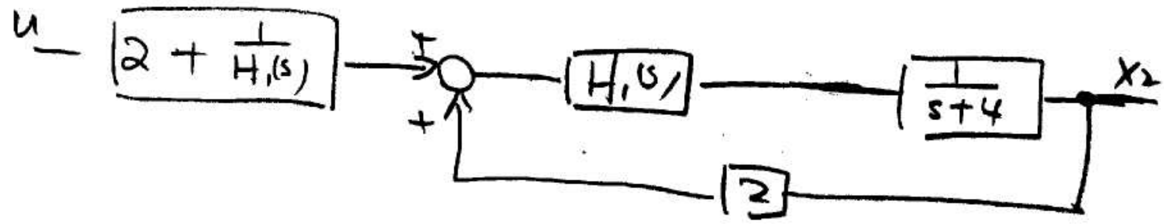
$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \theta_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{J_1} & \frac{-k_t k_b}{J_1 R} & \frac{k_s}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{J_2} & 0 & \frac{-k_s}{J_2} & \frac{-B}{J_2} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_t k_a}{J_1 R} \\ 0 \\ 0 \end{bmatrix} V_i$$

$$(y) = \begin{bmatrix} \frac{k_s}{J_2} & 0 & \frac{-k_s}{J_2} & \frac{-B}{J_2} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix}$$

(2)



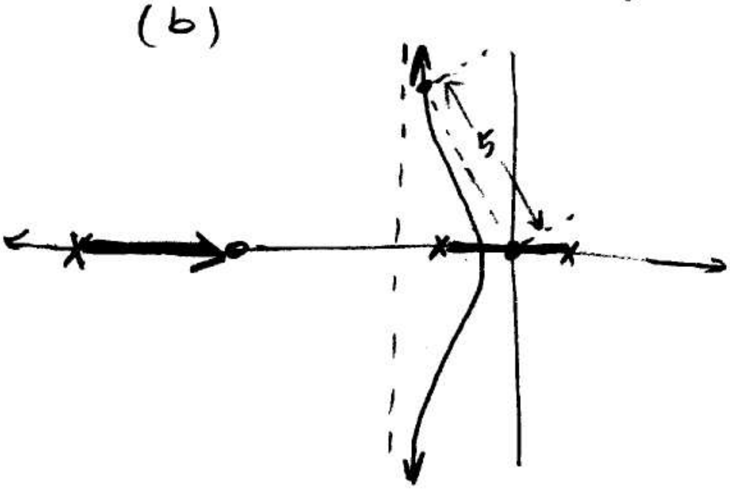




$$u \longrightarrow \boxed{\frac{s^2+s+10}{s^3+5s^2+6s}} \longrightarrow X_2$$

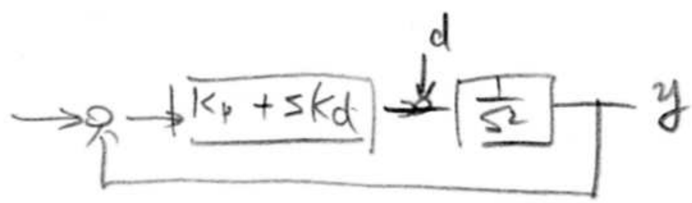
(3) (a) use Routh array. $\boxed{K > 8}$

(b)



$K \approx 40$

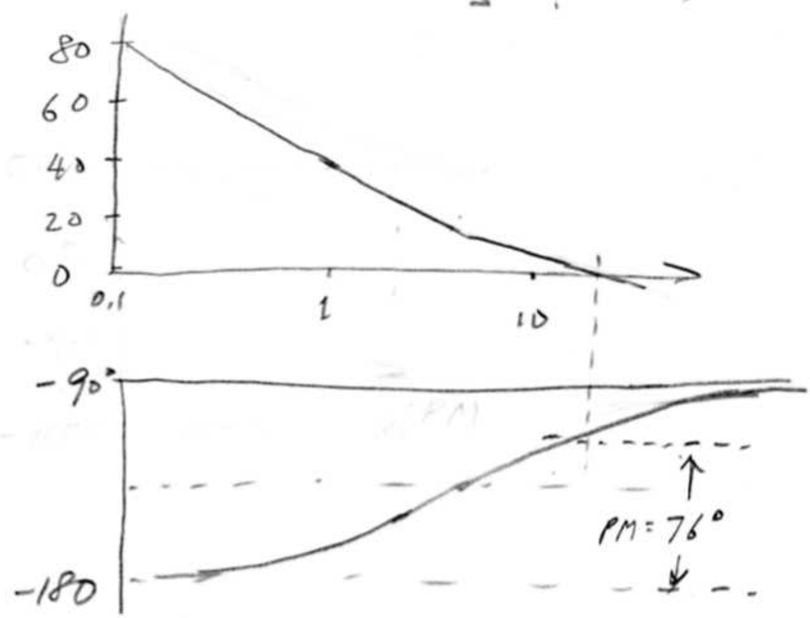
(4)



$$\frac{Y(s)}{d(s)} = \frac{1}{s^2 + k_p + sk_d}\Bigg|_{s=0} = 0.01 \Rightarrow k_p = 100$$

For double pole $k_d = 20$. ch. poly $= (s+10)^2$

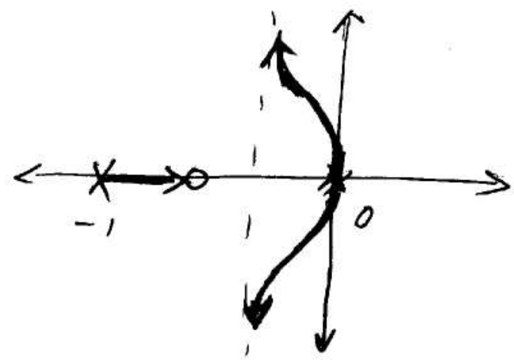
loop transfer function $\dfrac{20s+100}{s^2}$



(5) For a type II system, $C(s)$ must have 2 poles at the origin, i.e.,

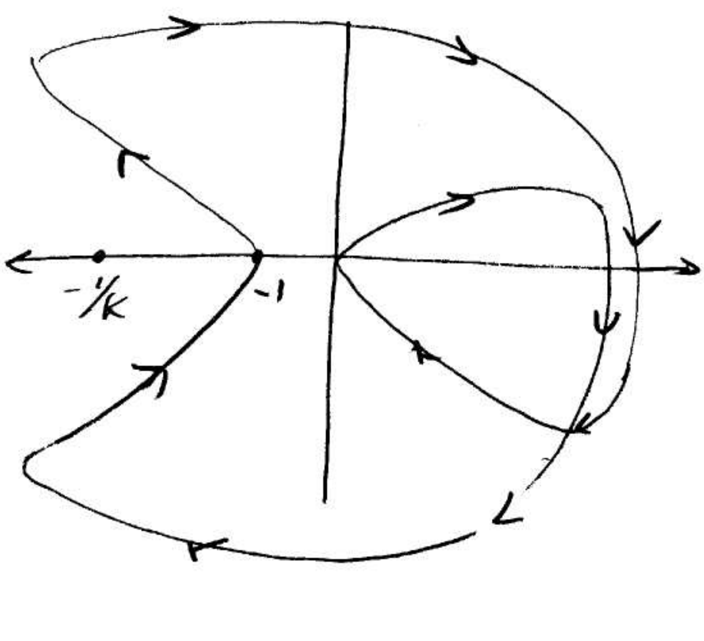
$$C(s) = \frac{k}{s^2}.$$

However, the system is unstable with this $C(s)$.
To stablize the system, we need to include a zero between [-1, 0).



For moving the asymptotes further to the left (for faster response), the zero should be placed closer to the origin. On the other hand, if the zero is close to the origin, a close-loop pole will move toward it (and slows down the response) as k increases.

(6)



For $0 < K < 1$, the system is stable.
For $K > 1$, there is a RHP pole.
For $K < 0$, there are 2 RHP poles