1. Orthogonal LLSE

(a) Consider zero-mean random variables $X, Y, Z$ such that $Y, Z$ are orthogonal. Show that $L[X \mid Y, Z] = L[X \mid Y] + L[X \mid Z]$.

(b) Show that for any zero-mean random variables $X, Y, Z$ it holds that:

$$L[X \mid Y, Z] = L[X \mid Y] + L[X \mid Z - L[Z \mid Y]]$$

2. Gaussian Estimation

Let $Y = X + Z$ and $U = X - Z$, where $X$ and $Z$ are i.i.d. $N(0, 1)$.

(a) Find the joint distribution of $U$ and $Y$. 
(b) Find the MMSE of $X$ given the observation $Y$, call this $\hat{X}(Y)$.

(c) Let the estimation error $E = X - \hat{X}(Y)$. Find the conditional distribution of $E$ given $Y$.

3. Joint Gaussian Probability

Let $X \sim \mathcal{N}(1, 1)$ and $Y \sim \mathcal{N}(0, 1)$ be jointly Gaussian with covariance $\rho$. What is $\mathbb{P}(X > Y)$?