

University of California - Berkeley
Department of Electrical Engineering & Computer Sciences
EE126 Probability and Random Processes
(Spring 2012)

Homework 3
Due Tuesday, February 7th

1. The hats of n persons are thrown into a box. The persons then pick up their hats at random (i.e., so that every assignment of the hats to the persons is equally likely). What is the probability that
 - (a) every person gets his or her hat back?
 - (b) the first m persons who picked hats get their own hats back?
 - (c) everyone among the first m persons to pick up the hats gets back a hat belonging to one of the last m persons to pick up the hats?

Now assume, in addition, that every hat thrown into the box has probability p of getting dirty (independently of what happens to the other hats or who has dropped or picked it up). What is the probability that

- (d) the first m persons will pick up clean hats?
 - (e) exactly m persons will pick up clean hats?
2. Alice plays with Bob the following game. First Alice randomly chooses 4 cards out of a 52-card deck, memorizes them, and places them back into the deck. Then Bob randomly chooses 8 cards out of the same deck. Alice wins if Bob's cards include all cards selected by her. What is the probability of this happening?
3. (a) Let X be a random variable that takes nonnegative integer values. Show that

$$\mathbf{E}[X] = \sum_{k=1}^{\infty} \mathbf{P}(X \geq k).$$

Hint: Express the right-hand side of the above formula as a double summation then interchange the order of the summations.

- (b) Use the formula in the previous part to find the expectation of a random variable Y whose PMF is defined as follows:

$$p_Y(y) = \frac{1}{b - a + 1}, \quad y = a, a + 1, \dots, b$$

where a and b are nonnegative integers with $b > a$. Note that for $y = a, a + 1, \dots, b$, $p_Y(y)$ does not depend explicitly on y since it is a uniform PMF.

4. Two fair three-sided dice are rolled simultaneously. Let X be the difference of the two rolls.
 - (a) Calculate the PMF, the expected value, and the variance of X .
 - (b) Calculate and plot the PMF of X^2 .

5. Let $n \geq 2$ be an integer. Show that

$$\sum_{k=2}^n k(k-1) \binom{n}{k} = n(n-1)2^{n-2}.$$

Hint: As one way of solving the problem, following from Example 1.31 in the text, think of a committee that includes a chair and a vice-chair.
