UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 2

Spring 2006

Issued: Thursday, January 26, 2006 **Due:** Thursday, February 2, 2006

Reading: Berstsekas & Tsitsiklis, §1.3, §1.4. and §1.5

Problem 2.1
A coin is tossed twice. Alice claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head. Is she right? Does it make a difference if the coin is fair or unfair?

Problem 2.2
An Englishman is driving down a road in the US. At time $t = 0$, he starts by driving by driving on the wrong side of the road with probability $p \in [0, 1]$. At each subsequent time instant $t = 1, 2, 3, \ldots$, he switches randomly with probability p to the other side of the road (from the side that he was on at time $t - 1$). Let P_t be the probability that he is on the correct side of the road at time t.

(a) Find a recursion for P_t in terms of P_{t-1} and p.

(b) Show that $P_t = \frac{1}{2}\left[1 + (1 - 2p)^{t+1}\right]$.

Problem 2.3
Larry and Consuela love to challenge each other to coin flipping contests. On one particular day, Larry notices that he has brought $2n + 1$ fair coins. Assume that he lets Consuela play with the $n + 1$ coins, and he plays with n coins. Show that the probability that after all the coins have been flipped Consuela will have gotten more heads than Larry is $\frac{1}{2}$.

Problem 2.4
In the kitchen in your apartment, you put all your 10 forks in the left drawer and all 10 knives in the right drawer. Your roommate, who does not agree with your organizational approach, comes in, takes two forks from the left drawer and tosses them into the right drawer. She then takes at random an item (knife or fork) from the right drawer and tosses it in the left drawer. After this exchange, you come in and randomly pick up an item from a randomly chosen drawer. Given you have picked up a knife, what is the probability that you have opened the left drawer?

Problem 2.5
Recall the "Monty Hall" problem from class and discussion (see also Example 1.12, p. 27). Consider the following variation of it. There are four doors, one and only one of which conceals a prize. You pick a door and the game-show host opens one of the remaining doors that does not conceal a prize. You are given the choice of sticking to your original decision or switching to either of the other two unopened doors.

(a) Let the door you originally picked be door 1. What is the probability that the prize is behind door 4?

(b) Suppose you picked 1, and the host opened door 2. What is the conditional probability that the prize is behind door 4? What is the conditional probability that the prize is behind door 1?

(c) Whether you switch or not, the host opens *another* door that does not have the prize behind it (so now two doors have been opened, one of which could be your initial choice if you switched). You are again offered the option to switch.

Should you switch or keep your choice in the two cases that you are given a choice, if your objective is to maximize the probability of winning?

Problem 2.6

Three persons roll a fair n-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12}, A_{13}, and A_{23} are pairwise independent but are not independent.

Problem 2.7

Anne, Betty, Chloe and Daisy were all friends in school. Subsequently each of the six subpairs meet up once; at each of the six meetings, the pair quarrels with some fixed probability p and otherwise the pair retains a firm friendship. Quarrels take place independently of each other. In the future, if any one of the four hears a rumour, she tells it to her firm friends only. Supposing that Anne hears a rumour, what is the probability that:

(a) Daisy hears it?

(b) Daisy hears it if Anne and Betty have quarrelled?

(c) Daisy hears it if Betty and Chloe have quarrelled?

(d) Daisy hears it if she has quarrelled with Anne?