UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Instructions and review for Midterm #1
Spring 2006

Exam format

- The first midterm will be held in class on Thursday, February 23. You are permitted to bring a calculator, and one $8.5 \times 11''$ sheet of notes (one side ONLY).
- The exam will cover all material from Chapters 1 and 2, as discussed in lectures #1 through #10, and covered in homeworks #1 through #4.

Review problems

Problem 5.1
On any given day your golf score is any integer from 101 to 110, each with probability 0.1. Determined to improve your score, you decide to play with three balls and declare as your score the minimum Y of the scores X_1, X_2, and X_3 obtained with balls 1, 2, and 3, respectively.

Calculate the PMF of Y. By how much has your expected score improved as a result of using three balls?

Problem 5.2

Suppose you and your friend play a game where each of you throws a 6-sided die, each of your throws being independent. Each time you play the game, if the largest of the two values you obtained from each die is greater than 4, then you win 1 dollar; otherwise, you lose 1 dollar. Suppose that you play the game $n \geq 3$ times, each game being independent of the others.

(a) What is the amount of money you expect to win on the first and last game combined?

(b) How much do you expect to win in your last game given that you lost in the first game?

(c) How much do you expect to have won in your last game given that you won the first game and you won a total of m dollars at the end?

(d) What is the probability that you won both the first and last game given that you won a total of m dollars at the end?

Problem 5.3
Consider the following game. You throw two fair coins independently. Let's refer to the coins as coin 1 and coin 2. If they have the same outcome, you win; otherwise, you lose.

(a) Is the outcome of coin 1 independent of whether you win or lose? Is the outcome of coin 2 independent of whether you win or lose? Explain your answer.

(b) Are the outcomes of the two coins independent of each other conditioned on your having won or lost? Explain your answer.

Problem 5.4

Your computer has been acting very strangely lately, and you suspect that it might have a virus on it. Unfortunately, all 12 of the different virus detection programs you own are somewhat outdated. You know that if your computer really does have a virus, each of a programs, independently of the others, has a .8 chance of correctly identifying your computer to be infected, and a .2 chance of thinking your computer is fine. On the other hand, if your computer does not have a virus, each program has a .9 chance of believing you computer to be free from viruses, and a .1 chance of wrongly thinking your computer is infected. Given that your computer has a 65% chance of being infected with some virus, and given that you will believe your virus protection programs only if 9 or more of them agree, find the probability that your detection programs will lead you to the right answer.

Problem 5.5

Your filthy roommate, Jerome, is (finally) doing his laundry. He has so much to do that he is doing it over four separate days—Monday, Tuesday, Wednesday, and Thursday—and, on each day, he has five quarters in each of his left and right pockets for use in the coin-operated machines. When drawing a quarter from his pockets, he draws from his left pocket with probability p and from his right pocket with probability $1 - p$, independently of previous draws.

On Monday, Jerome draws five quarters from his pockets to get the \$1.25 required for a washing machine. It turns out that one quarter is drawn from his left pocket and four quarters are drawn from his right pocket. To begin with, one of the quarters in his left pocket is Canadian (and all the other nine are American).

(a) What is the probability that his Canadian quarter remains in his left pocket?

(b) What is the expectation and variance of the number of American quarters remaining in his left pocket?

(c) On Tuesday, Jerome again draws five quarters from his pockets. What is the probability that exactly three of the five quarters drawn are from his left pocket?

(d) On Wednesday, Jerome decides to use the extra-large capacity washing machine, which costs \$2.00; so he draws eight quarters from his pockets. It turns out that the eighth quarter drawn is the fifth drawn from his left pocket, leaving it empty. What is probability of this event?

(e) On Thursday, Jerome does not draw a fixed number of quarters from his pockets, but instead draws quarters until one of his pockets is empty. Let X be the number of quarters remaining in his other pocket. Let A be the event that Jerome's left pocket empties before his right pocket. Find $\mathbb{P}(\{X = x\} \cap A)$ for $x = 1, 2, \ldots, 5$.