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UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
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Professor Tse Fall 1998

EECS 126 — MIDTERM #2

6 November 1998, 11:10–12:10

[20 pts.] 1a. X, Y are two jointly distributed random variables. Are the following statements true in general? If so, explain. If not, give examples for which they are true.

i) $E(X+Y) = E(X) + E(Y)$

ii) $E(XY) = E(X) \cdot E(Y)$

iii) $E\left(\frac{X}{Y}\right) = \frac{E(X)}{E(Y)}$

iv) $E(XY \mid Y = 3) = 3E(X)$

[15 pts.] 1b:

i) If X and Y are independent, is $E(Y|X) = E(Y)$? Explain.

ii) Suppose now

$$Y = \begin{cases} X & \text{with probability } \frac{1}{2} \\ -X & \text{with probability } \frac{1}{2} \end{cases}$$

Is $E(Y|X) = E(Y)$? Is X, Y independent?

[30 pts.] 2. Starting from the origin, a particle takes a random walk in the plane. Each step is of unit length and is equally likely in any direction, and the direction taken in each step is independent of any other. Find the expected **squared distance** from the origin after n steps.

[35 pts.] 3. A message containing a random number M bits is transmitted through a binary symmetric channel with cross-over probability ε. M is geometrically distributed with pariameter p (i.e., $P(M = k) = (1-p)^k p, \quad k \geq 0$). Let N be the total number of bits transmitted through correctly. Find:

a. $E(N)$ **(10 pts.)**

b. $Var(N)$ **(15 pts.)**

c. pmf of N **(10 pts.)**

(The mean and variance of M are $\frac{1-p}{p}$ and $\frac{1-p}{p^2}$, respectively.)