Problem 1. Conditioning on the Minimum of Uniforms
If $X$ and $Y$ are independent $Uniform[0,1]$, show that
\[ E(Y \mid \min\{X,Y\}) = \frac{1}{4} + \frac{3}{4} \min\{X,Y\}. \]

Problem 2. MMSE with Balls in Bins
We throw $n \geq 1$ balls into $m \geq 2$ bins. Let $X$ and $Y$ represent the number of balls that land in bin 1 and 2 respectively.

1. Calculate $E[Y \mid X]$.
2. What are $L[Y \mid X]$ and $Q[Y \mid X]$ (where $Q[Y \mid X]$ is the best quadratic estimator of $Y$ given $X$)?
   
   *Hint:* Your justification should be no more than two or three sentences, no calculations necessary! Think carefully about the meaning of the MMSE.

Problem 3. MMSE for Jointly Gaussian
Let $[X \ Y \ Z]^T \sim N(\mu, \Sigma)$, and
\[ \mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]
and
\[ \Sigma = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 2 \end{bmatrix}. \]

Find $E[X \mid Y, Z]$. 