1. Uncorrelated & Independent

(a) If $X$ and $Y$ are uncorrelated, $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$.

(b) If $X_1, \ldots, X_n$ are uncorrelated, $\text{var}(X_1 + \cdots + X_n) = \sum_{i=1}^{n} \text{var}(X_i)$.

(c) Show that independent random variables are uncorrelated.

(d) Find an example, where a pair of random variables are uncorrelated but not independent.

2. Second Moment Method

Consider a non-negative RV $Y$, with $\mathbb{E}(Y^2) < \infty$. Show that

$$\mathbb{P}(Y > 0) \geq \frac{\mathbb{E}(Y)^2}{\mathbb{E}(Y^2)}.$$ 

Hint: Use Cauchy-Schwarz on $Y \mathbf{1}_{\{Y > 0\}}$.

3. Conditioning on the Minimum of Uniforms

If $X$ and $Y$ are independent Uniform$[0,1]$, show that

$$\mathbb{E}(Y \mid \text{min}\{X, Y\}) = \frac{1}{4} + \frac{3}{4} \text{min}\{X, Y\}.$$