1. MMSE from Joint Density
   Let the joint density of two random variables $X$ and $Y$ be
   \[ f_{X,Y}(x,y) = \frac{1}{4}(2x+y)1\{0 \leq x \leq 1\}1\{0 \leq y \leq 2\}. \]
   First show that this is a valid joint distribution. Suppose you observe $Y$ drawn from this joint density. Find $\text{MMSE}[X \mid Y]$.

2. MMSE for Jointly Gaussian
   Let $[X \ Y \ Z]^T \sim \mathcal{N}(\mu, \Sigma)$, and
   \[
   \mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
   \]
   and
   \[
   \Sigma = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 2 \end{bmatrix}.
   \]
   Find $\mathbb{E}[X \mid Y,Z]$.

3. Recursive JG MMSE
   Let $(V_n, n \in \mathbb{N})$ be i.i.d. $\mathcal{N}(0, \sigma^2)$ and independent of $X_0 = \mathcal{N}(0, \sigma^2)$. Define
   \[ X_{n+1} = aX_n + V_n, \quad n \in \mathbb{N}. \]
   (a) What is the distribution of $X_n$, where $n$ is a positive integer?
   (b) Find $\mathbb{E}[X_{n+m} \mid X_n]$ for $m, n \in \mathbb{N}$, $m \geq 1$.
   (c) Find $u$ so that the distribution of $X_n$ is the same for all $n \in \mathbb{N}$. 