Probability in EECS

Jean Walrand – EECS – UC Berkeley

Probability in EECS - Fall 2013

Topics:
PageRank, Multiplexing, Digital Links, Speech Recognition, Path Planning, Tracking, Complements.

This course explains important applications of probability in EE and CS. The emphasis is on acquiring a working knowledge of the methodology. Homework assignments consist of Matlab experiments and problems.

Course Organization:
Lectures, one hour of discussion, two midterms (20% each), final (35%), hw (25%). We use self-grading of HW.

Prerequisites:
CS70, Math 54

Texts:
Bertsekas and Tsitsiklis: Introduction to Probability Theory;
Lecture notes provided.

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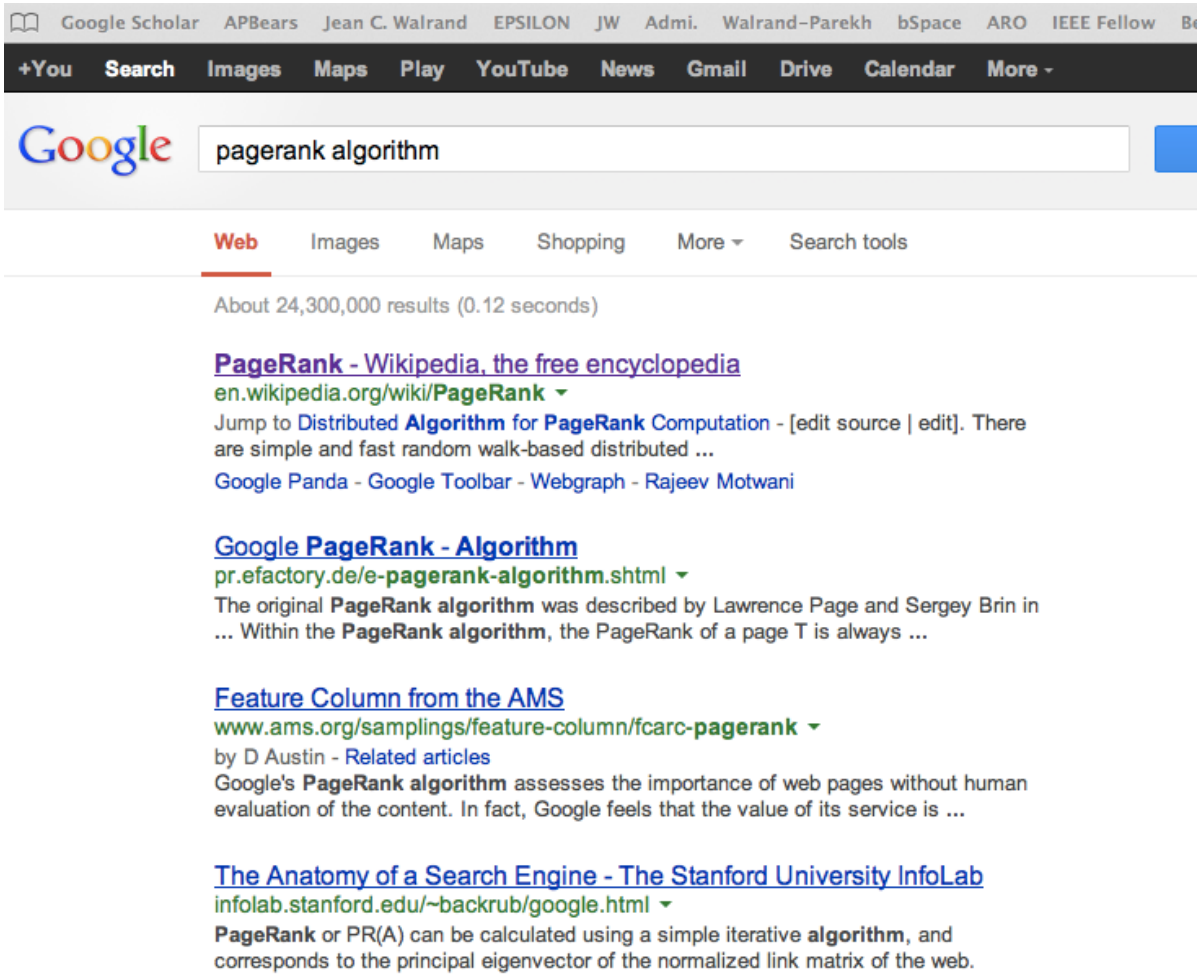
PageRank: How to rank search results



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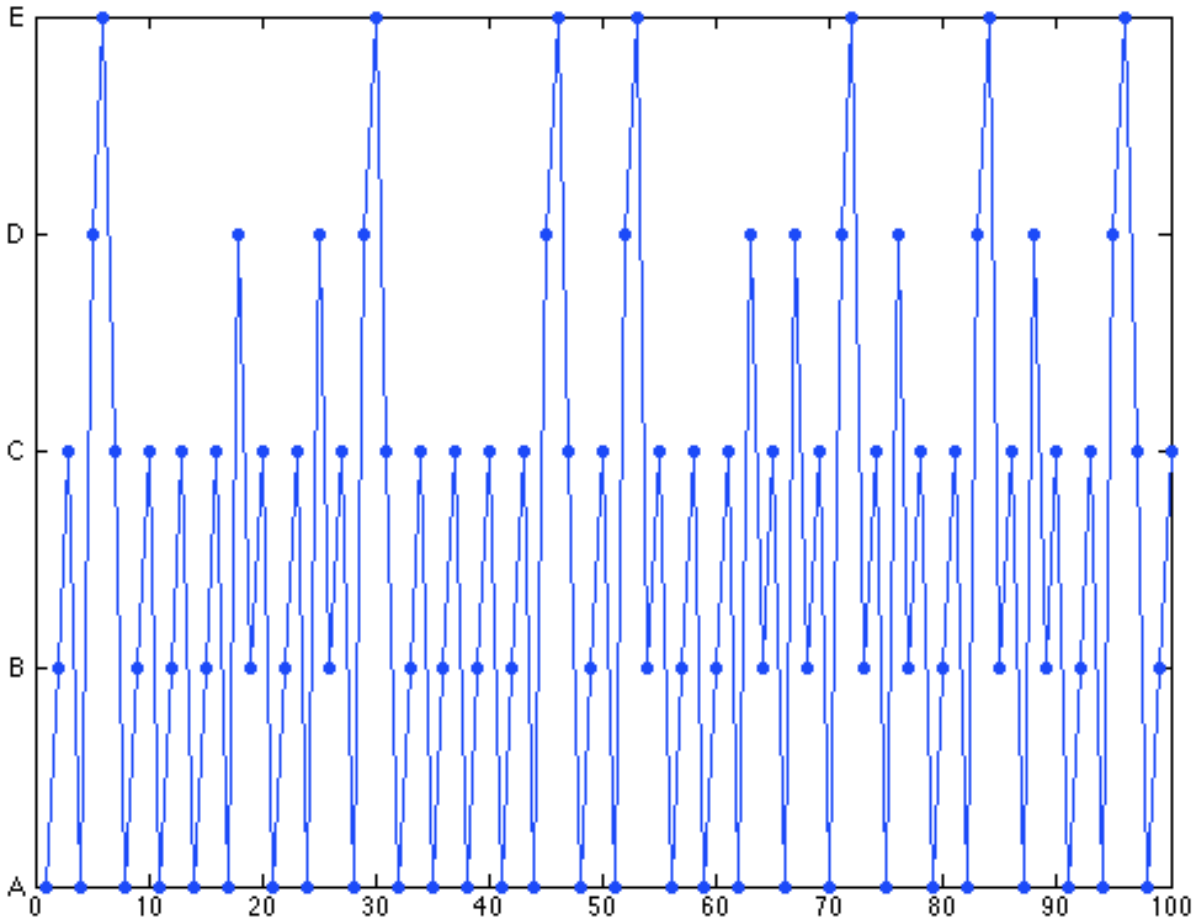
PageRank: Markov Chains



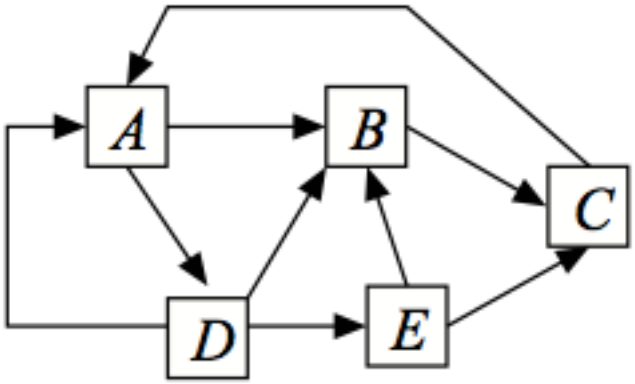


Figure 1.2: Andrey Markov. 1856-1922.

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Multiplexing

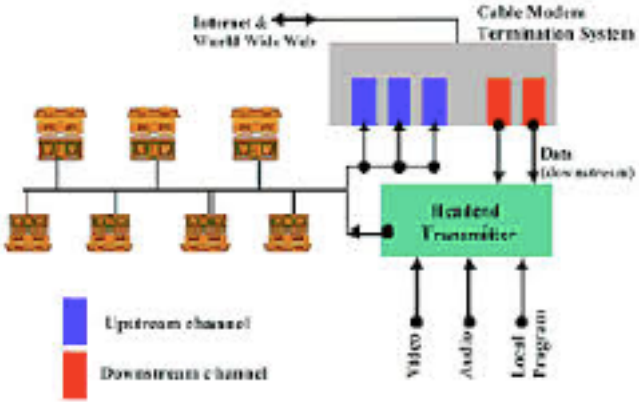

Figure 2.1: Shared coaxial cable for internet access.

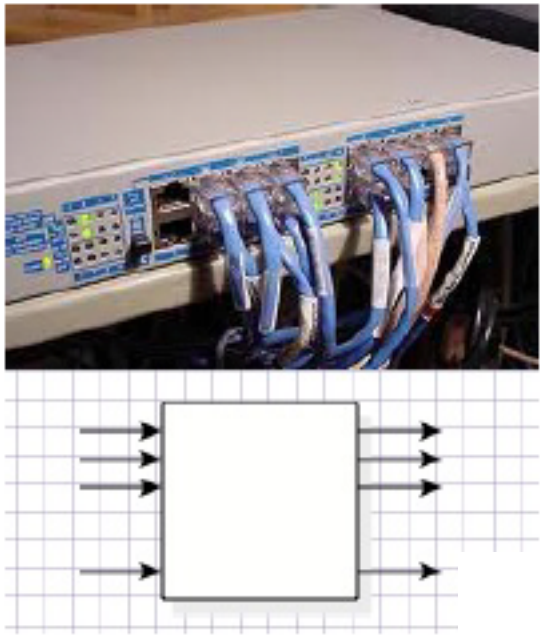

Figure 2.11: A switch with multipl input and output ports

Figure 2.16: A number of smartphones share a WiFi access point.

Figure 2.17: Norman Abramson, b. 1932.

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Digital Link

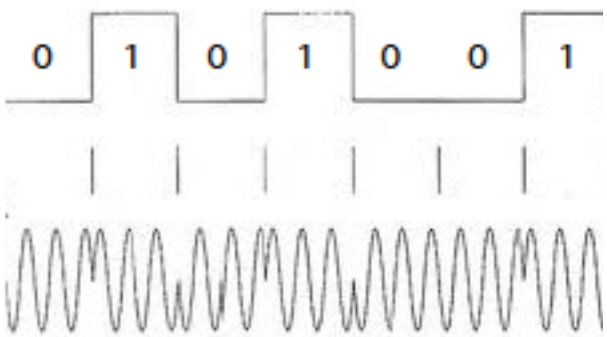

Figure 3.6: The signal that the transmitter sends when using BPSK.

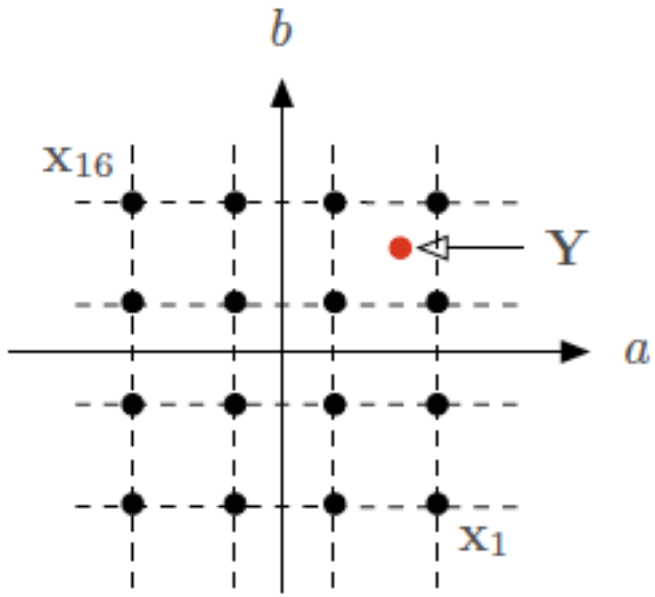

Figure 3.7: A QAM-16 constellation.

Figure 2.9: Carl Friedrich Gauss. 1777-1855

Low speed, high noise DSL, Cable Modems, etc.

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Tracking

Figure 4.2: Estimating the location of a device from satellite signals.

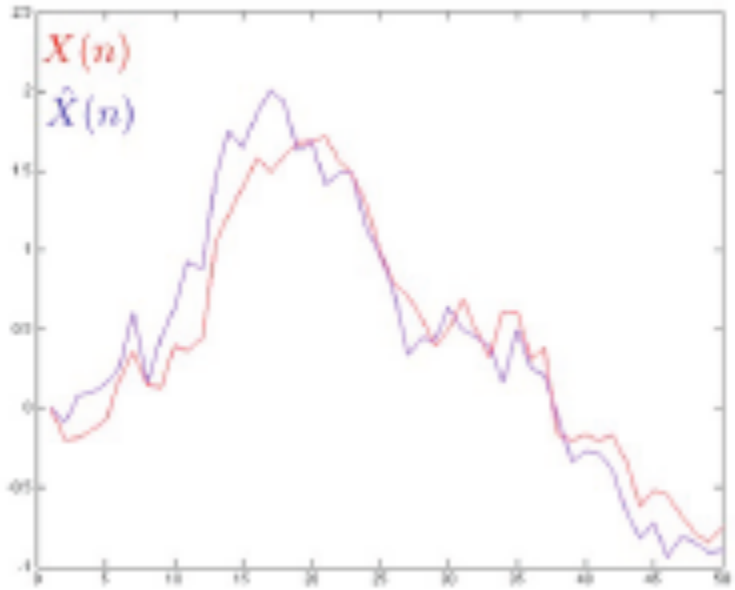

Figure 4.14: Rudolf Kalman, b. 1930.

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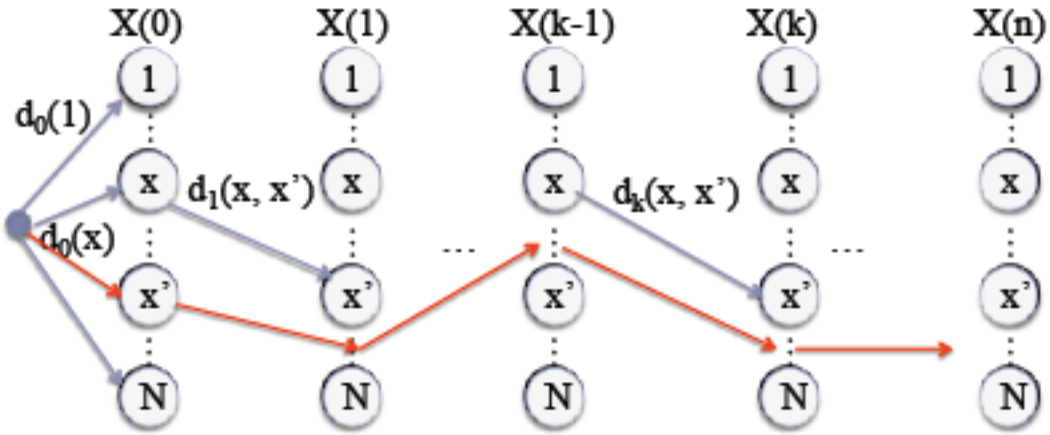
Speech Recognition

Figure 5.1: Can you hear me?



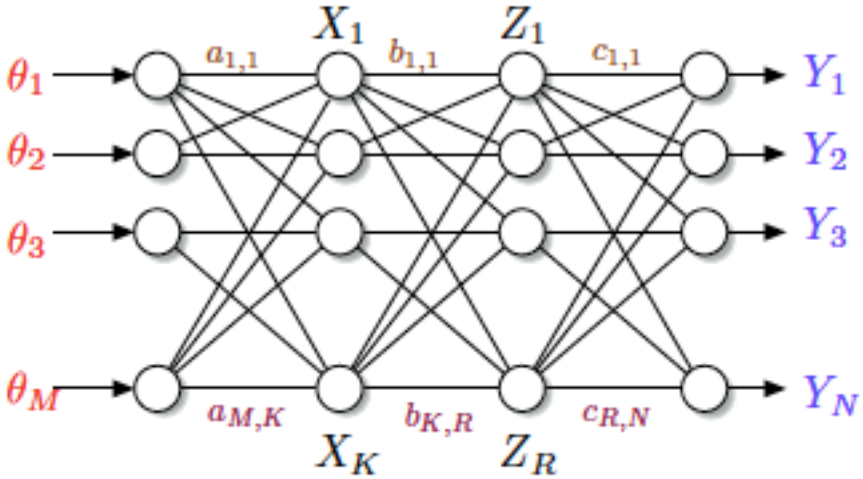


Figure 5.5: Andrew Viterbi, b. 1934.

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Path Planning

Figure 6.1: Road network. How to select a path?

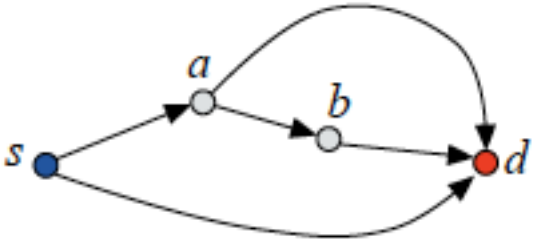

Figure 6.2: A simple graph.

Figure 5.4: Richard Bellman, 1920-1984.

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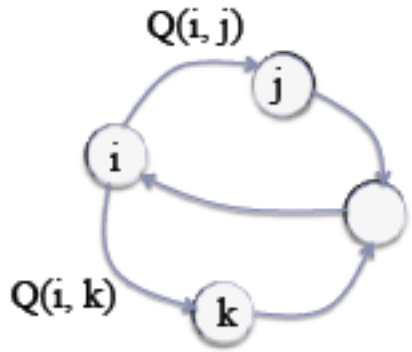
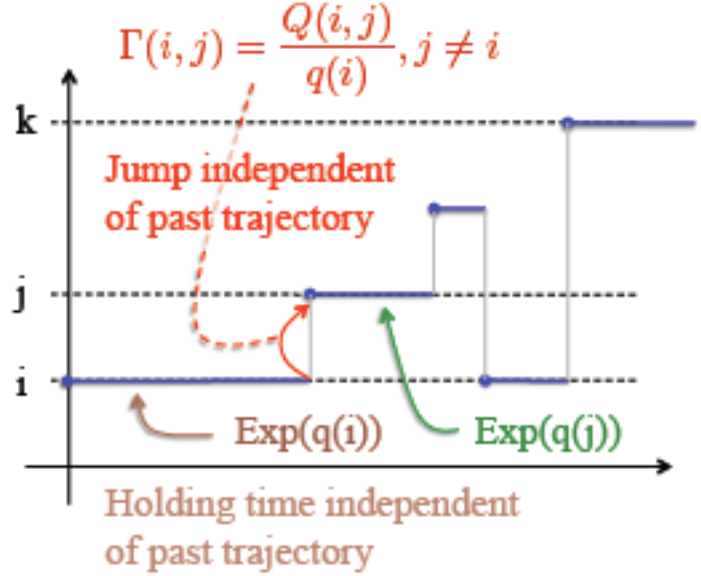
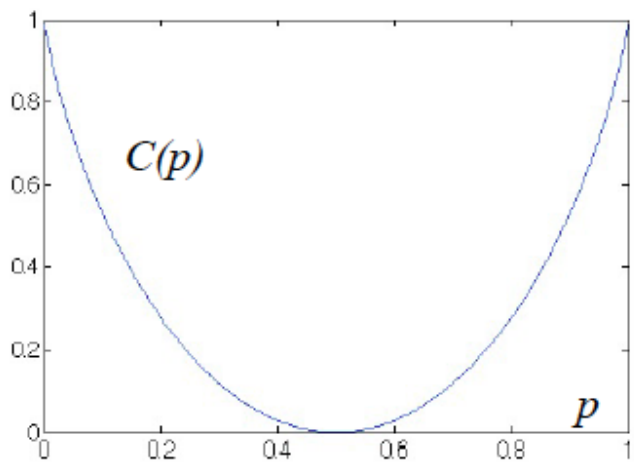
Complements







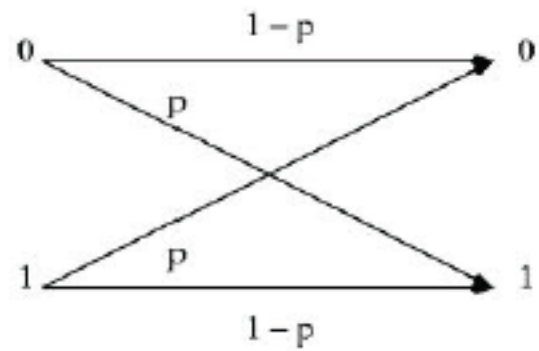


Figure 3.4: The binary symmetric channel.

Figure 7.10: Claude Shannon. 1916-2001.

Probability in EECS - Fall 2013 - Notes

Date	Topic	Assignments	Notes	Book
8/29	PageRank	H1	1.1-3, A.1-2	1, 2
9/3-5	PageRank	H2	1.4-5	7.1-2, 7.
9/10-12	PageRank	H3	1.6	5.1-3, 5.
9/17-19	Multiplexing	H4	2.1-4, A.3	5.4
9/24-26	Multiplexing		2.5-6	7.3
10/1-3	M1 - Link	H5	3.1-2	8.1-2
10/8-10	Link	H6	3.3-6	9.3
10/15-17	Tracking	H7	4.1-3	8.3-4, 9.1-2
10/22-24	Tracking	H8	4.4-7	
10/29-31	Speech		5.1-3	
11/5-7	M2 – Speech	H9	5.4	
11/12-14	Planning	H10	6.1-4	
11/19-21	Planning	H11	6.5-6	
11/26-28	Complements	H12	7.1-3	
12/3-5	Complements	H13	7.4-6	
12/* Final				

Probability in EECS - Fall 2013 - Book

Introduction to Probability

SECOND EDITION

Dimitri P. Bertsekas and John N. Tsitsiklis

Massachusetts Institute of Technology

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Probability is common sense reduced to calculation

Laplace

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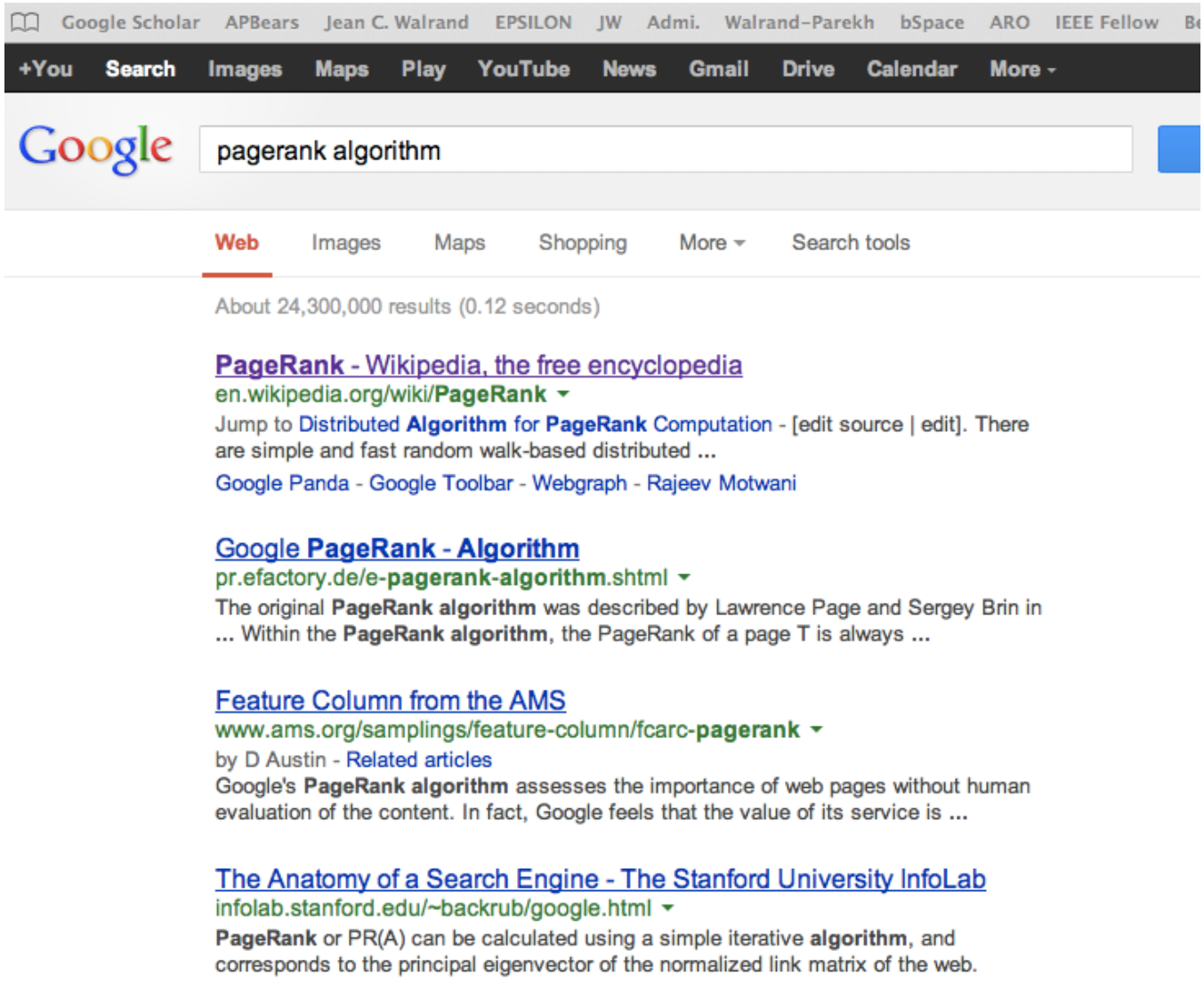
PageRank – Search Example

How to rank search results?



PageRank – Patent

How to rank search results?

United States Patent **6,285,999**
Page **September 4, 2001**

**Please see images for: (Certificate of Correction) **

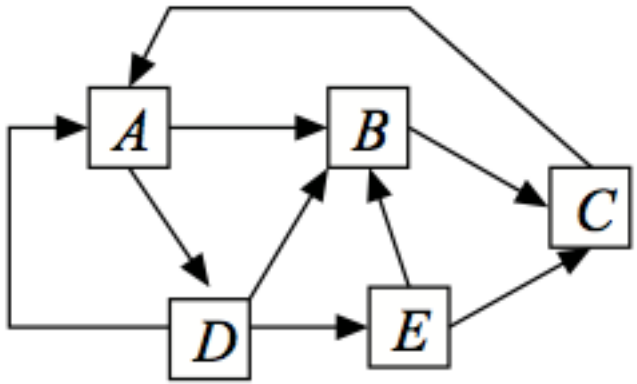
Method for node ranking in a linked database

Abstract

A method assigns importance ranks to nodes in a linked database, such as any database of documents containing citations, the world wide web or any other hypermedia database. The rank assigned to a document is calculated from the ranks of documents citing it. In addition, the rank of a document is calculated from a constant representing the probability that a browser through the database will randomly jump to the document. The method is particularly useful in enhancing the performance of search engine results for hypermedia databases, such as the world wide web, whose documents have a large variation in quality.

Inventors: **Page; Lawrence** (Stanford, CA)
Assignee: **The Board of Trustees of the Leland Stanford Junior University** (Stanford, CA)
Family ID: **26673538**
Appl. No.: **09/004,827**
Filed: **January 9, 1998**

PageRank – Example



$$\pi(A) = \pi(C) + \pi(D)(1/3)$$
$$\pi(B) = \pi(A)(1/2) + \pi(D)(1/3) + \pi(E)(1/2)$$
$$\pi(C) = \pi(B) + \pi(E)(1/2)$$
$$\pi(D) = \pi(A)(1/2)$$
$$\pi(E) = \pi(D)(1/3).$$

$$\pi = [\pi(A), \pi(B), \pi(C), \pi(D), \pi(E)] = \frac{1}{39}[12, 9, 10, 6, 2].$$

PageRank – Markov Chain

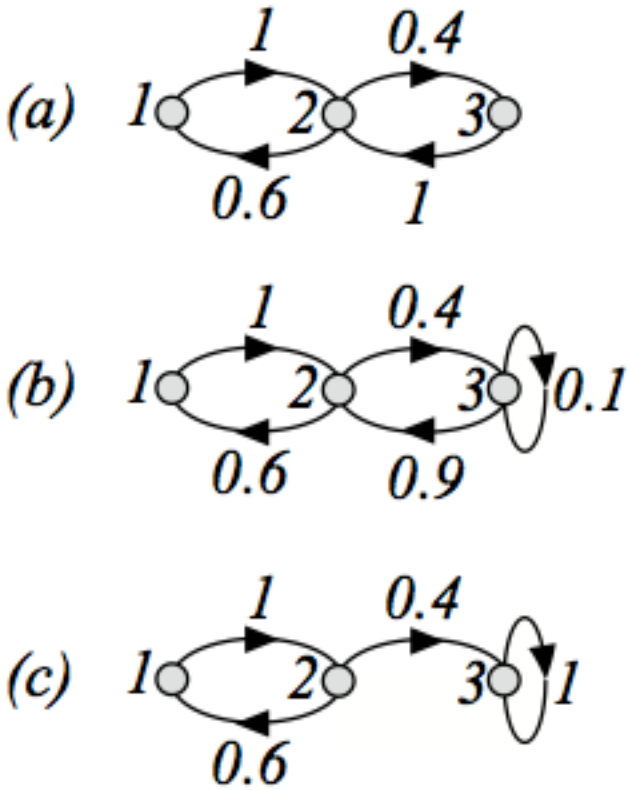
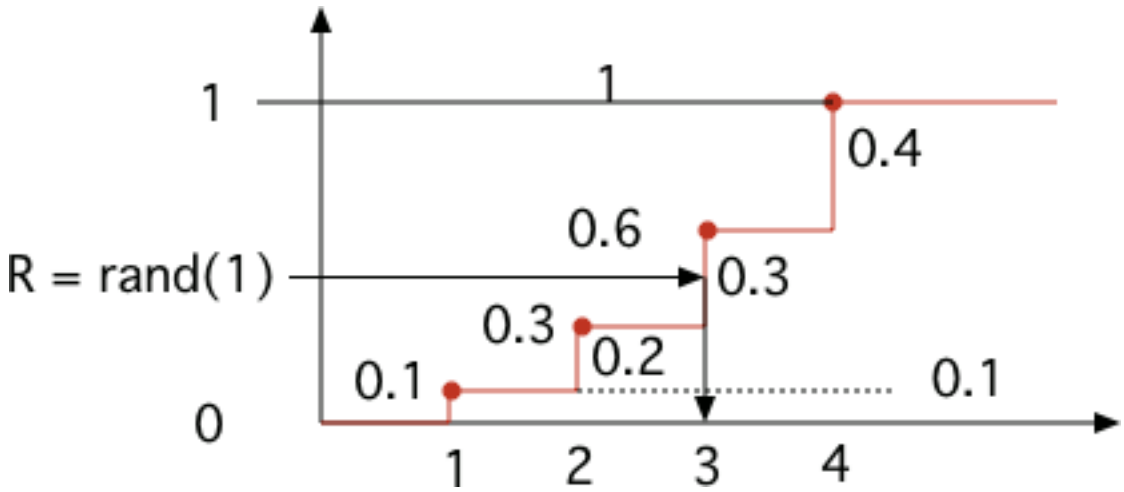


Figure 1.2: Andrey Markov. 1856-1922.

$$P[X(n+1) = j | X(n) = i, X(m), m < n] = P(i,j), \forall i,j \in \mathcal{X}, n \geq 0.$$

PageRank – Markov Chain - Simulation

Let X = k with probability P(k), k = 1, …, K.
How do we simulate X?
Example: P(1) = 0.1, P(2) = 0.2, P(3) = 0.3, P(4) = 0.4.



Pnorm = [0, 0.1, 0.2, 0.3, 0.4]
Pcum = cumsum(Pnorm) = [0, 0.1, 0.3, 0.6, 1]
R = rand(1) --> 0.52
[a, X] = histc(R, Pcum) --> a = [0, 0, 1, 0, 0], X = 3

PageRank – Markov Chain - Simulation

Let X = k with probability P(k), k = 1, …, K.
How do we simulate X?
Example: P(1) = 0.1, P(2) = 0.2, P(3) = 0.3, P(4) = 0.4.

```
function T = discrete(P)
% This function generates a
% random variable equal to k w.p. P(k)/sum(P).
 % Here, P = [P(1), P(2), ..., P(K)] where
% the P(k) are nonnegative.
Pnorm=[0 P]/sum(P);
Pcum=cumsum(Pnorm);
R=rand(1);
[~,T] = histc(R,Pcum);
```

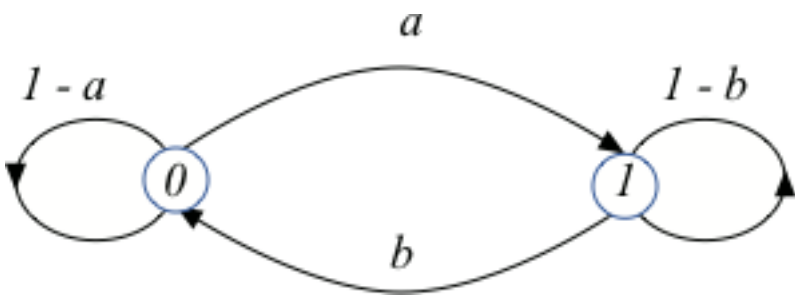
PageRank – Markov Chain - Simulation

```
M = 100;
A = 1;
P = [0, 0.3, 0.7; 0, 0.4, 0.6; 1, 0, 0];
X = zeros(1,M);
X(1) = A;
 for m = 1:M-1
      X(m+1) = discrete(P(X(m),:));
end
```

PageRank – Markov Chain - Simulation

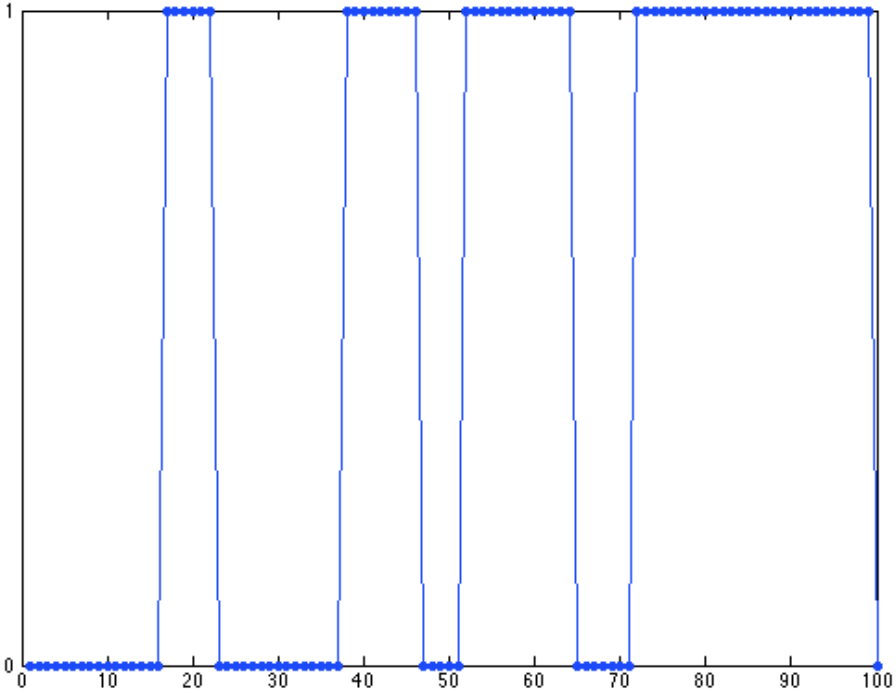
```
function simMC(M,A,P)
%simulate a MC for M steps with initial state A and tpm
P
%
X = zeros(1,M);
X(1) = A;
for m = 1:M-1
    X(m+1) = discrete(P(X(m),:));
end
plot(X,'.-b')
```

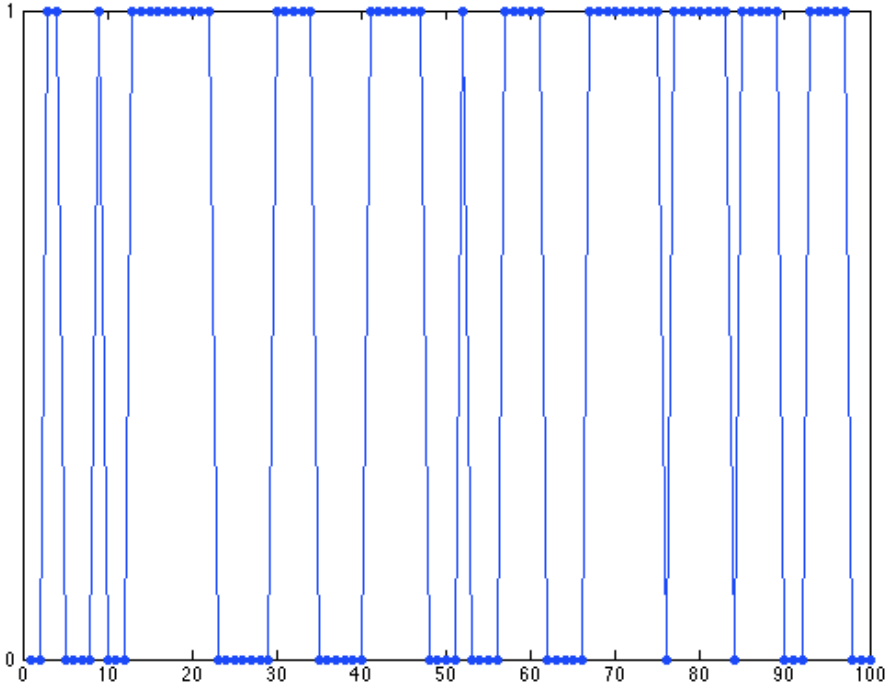
PageRank – Markov Chain - Simulation




```
>> M = 100; A = 1; P = [0.9, 0.1; 0.1, 0.9];
>> simMC(M,A,P)
```

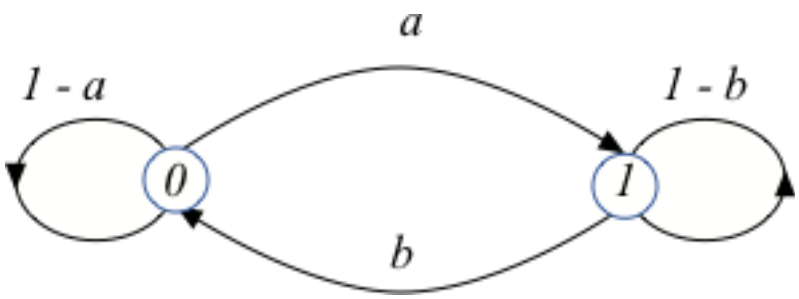
```
>> P = [0.7, 0.3; 0.3, 0.7];
>> simMC(M,A,P)
```



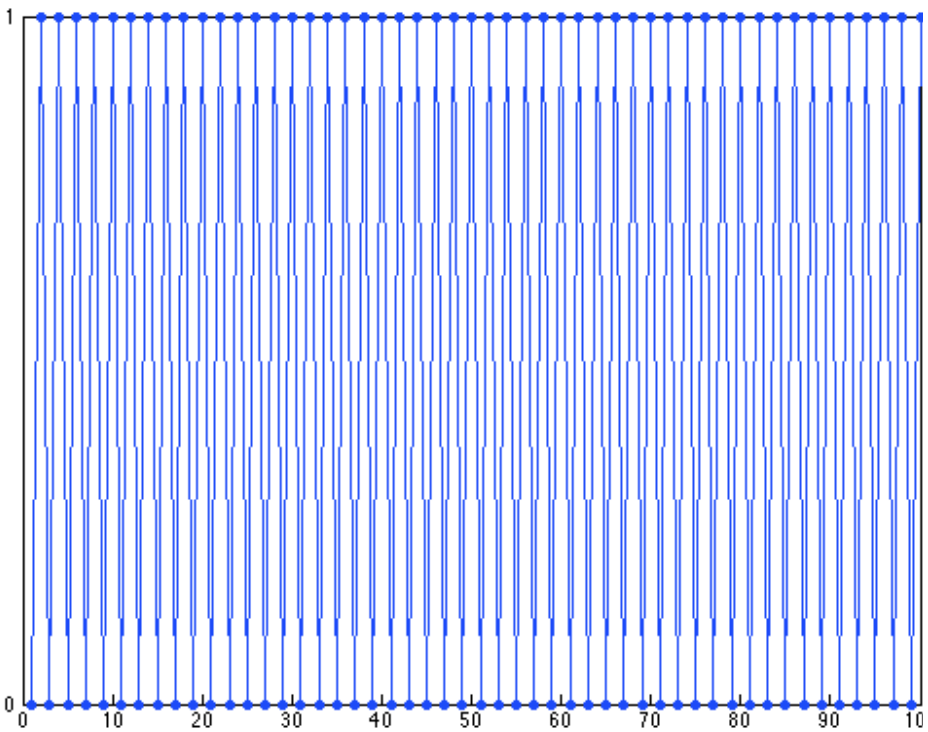


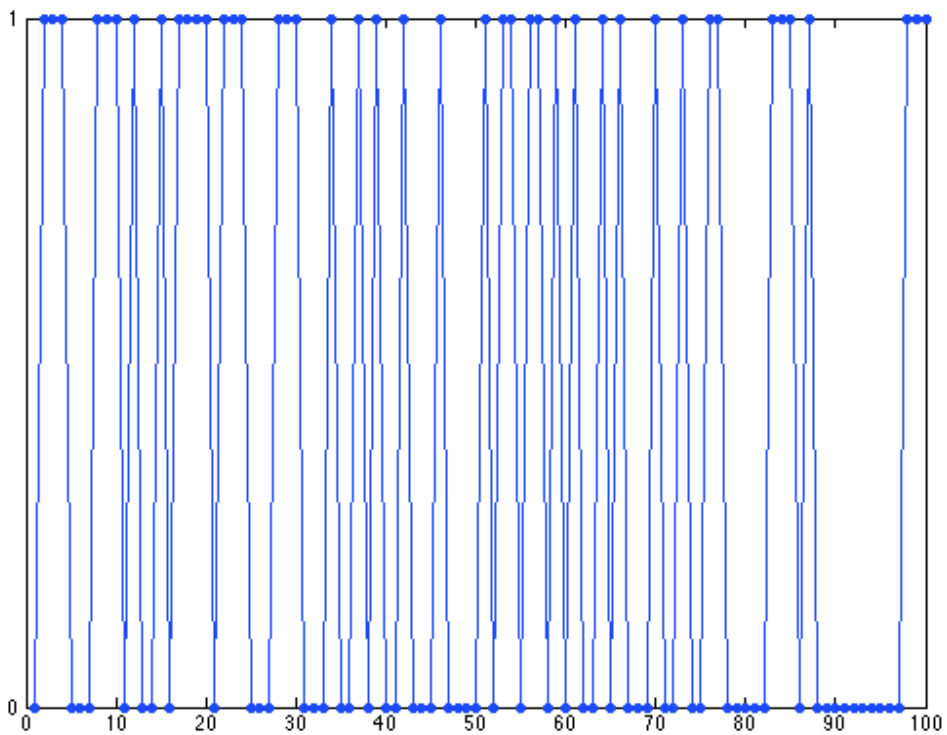
PageRank – Markov Chain - Simulation

```
>> P=[0,1;1,0];
>> simMC(M,A,P)
```



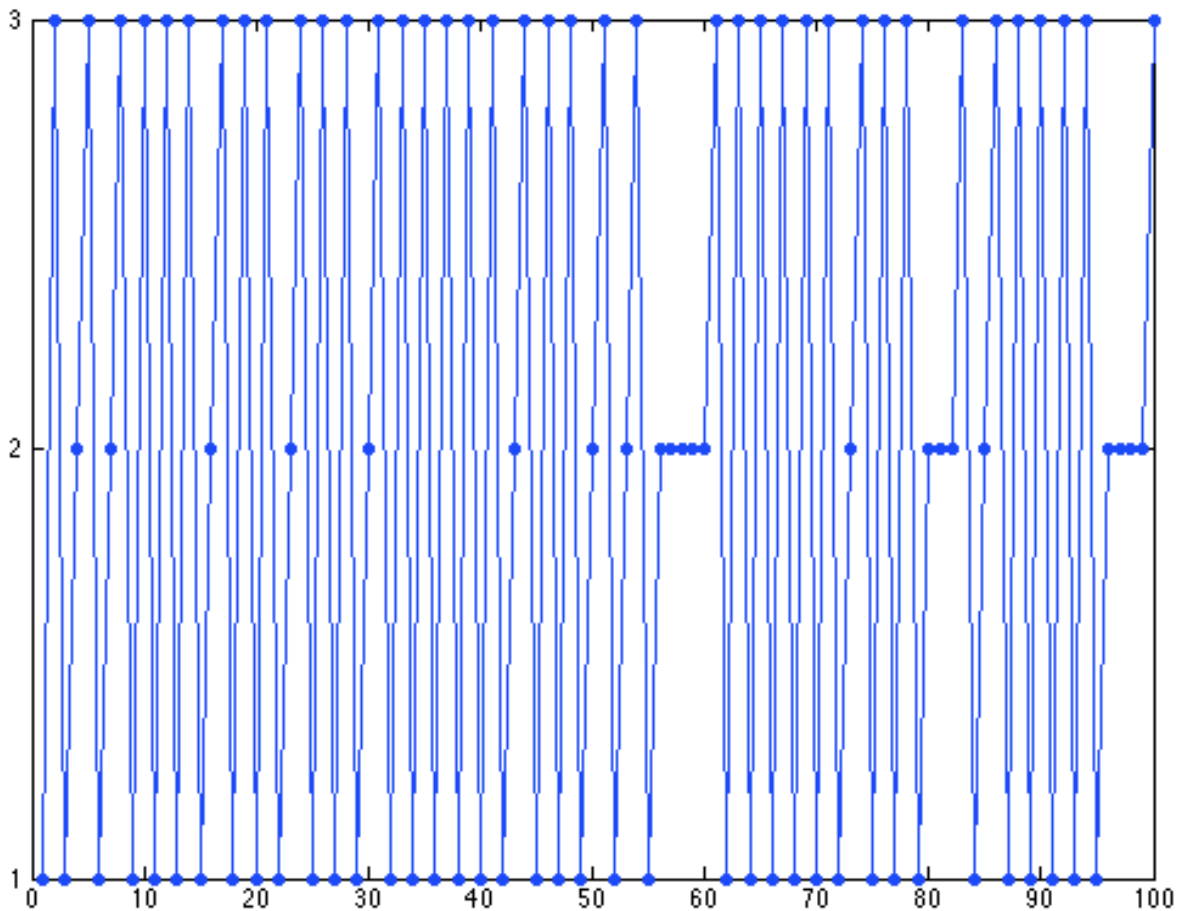

```
>> P=[0.5,0.5;0.5,0.5];
>> simMC(M,A,P)
```

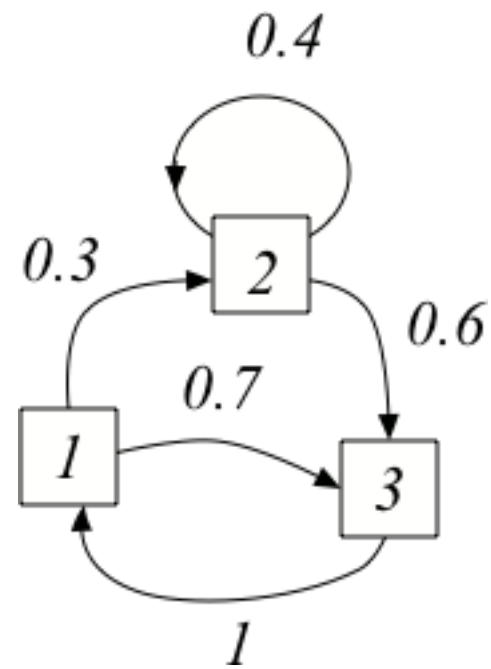




PageRank – Markov Chain - Simulation

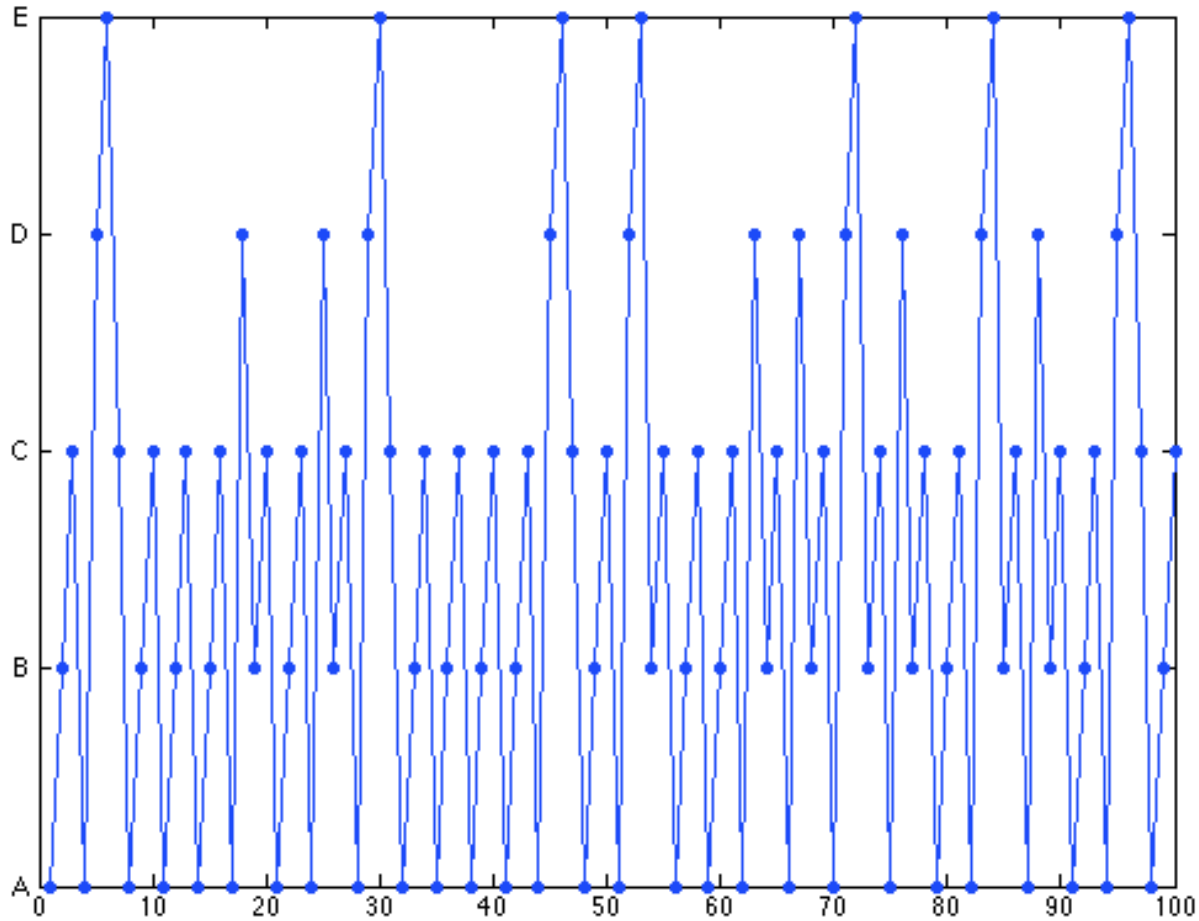
```
>> P=[0,0.3,0.7;0, 0.4,0.6;1,0,0];
>> simMC(M,A,P)
```

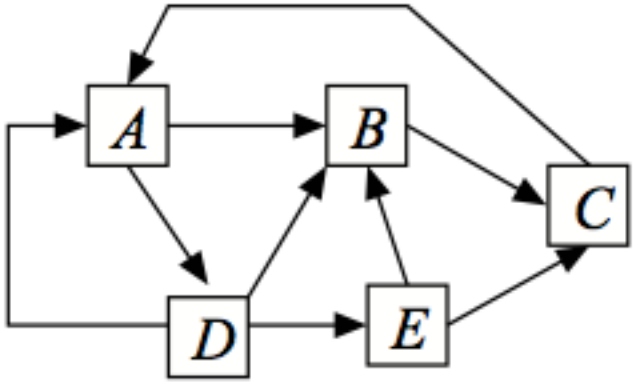




PageRank – Markov Chain - Simulation

```
P=[0,0.5,0,0.5,0;0,0,1,0,0;1,0,0,0,0;1/3,1/3,0,0,1/
3;0,0.5,0.5,0,0];
>> simMC(M,A,P)
```



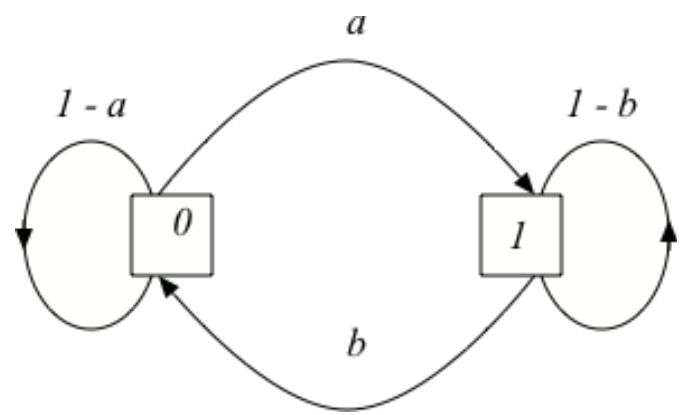


PageRank – Markov Chain – Inv. Dist.

Invariant Distribution

$$\pi = \pi P \leftarrow \text{Balance Equations}$$

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$



$$\pi(P - I) = 0$$

$$\pi \begin{bmatrix} -a & a \\ b & -b \end{bmatrix} = [0, 0] \qquad \pi \begin{bmatrix} -a & 1 \\ b & 1 \end{bmatrix} = [0, 1]$$

$$\pi = [0, 1] \begin{bmatrix} -a & 1 \\ b & 1 \end{bmatrix}^{-1} = [\frac{b}{a+b}, \frac{a}{a+b}]$$

PageRank – Markov Chain – Inv. Dist.

Invariant Distribution

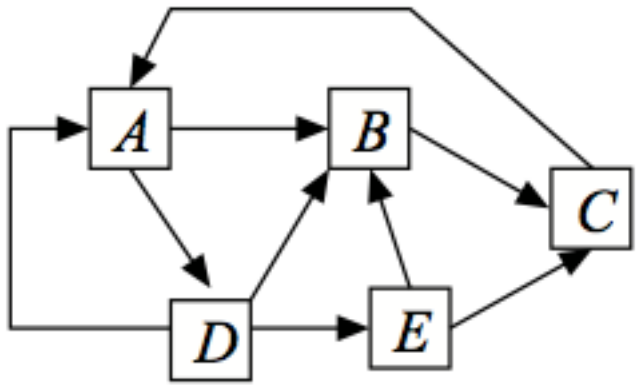
$$\pi = \pi P \leftarrow \text{Balance Equations}$$

$$\pi(P - I) = [0, \ldots, 0];$$

$$\pi B = [0, \ldots, 0, 1] \text{ where } B = [(P - I)_1, \ldots, (P - I)_{n-1}, \mathbf{1}]$$

$$\pi = [0, \ldots, 0, 1]B^{-1}$$

PageRank – Markov Chain – Inv. Dist.



π = [0.3077 0.2308 0.2564 0.1538 0.0513]