UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS126: Probability in EECS

Problem Set 6
Fall 2013

Issued: Tuesday, October 08, 2013 **Due:** In class Thursday, October 17, 2013

Problem 1. Simulate the following communication channel. There is an i.i.d. source that generates symbols $\{1, 2, 3, 4\}$ according to a prior distribution $\pi = [p_1, p_2, p_3, p_4]$. The symbols are modulated by QPSK scheme, i.e. they are mapped to constellation points $(\pm 1, \pm 1)$. The communication is on a baseband Gaussian channel, i.e. if the sent signal is (x_1, x_2), the received signal is

$$y_1 = x_1 + z_1,$$

$$y_2 = x_2 + z_2,$$

where z_1 and z_2 are independent $N(0, \sigma^2)$ random variables. Find the MAP detector and ML detector analytically. Simulate the channel using Simulink or MATLAB (Simulink is preferred.) for $\pi = [0.1, 0.2, 0.3, 0.4]$, and $\sigma = 0.1$ and $\sigma = 0.5$. Plot the error curve, which is a binary signal indicating whether the symbol is detected correctly or not.

Problem 2. Consider random variable Y which is exponentially distributed with parameter θ. You observe n i.i.d. samples of this random variable $y_1, \ldots, y_n$. Calculate the Maximum-likelihood estimator of θ. Let $\theta_{ML}(y_1, \ldots, y_n)$ be the ML estimator. What is the bias of this estimator, i.e. $E(\theta_{ML} - \theta|\theta)$? Does the bias converge to 0 as n goes to infinity?

Problem 3. Consider random variable Y which is uniform $u[a, b]$. You observe n i.i.d. samples of this random variable $y_1, \ldots, y_n$. Calculate the Maximum-likelihood estimator of a and b. What is the bias of this estimator?

Problem 4. Consider a hypothesis testing problem that if $X = 0$, you observe a sample of $N(\mu_0, \sigma^2)$, and if $X = 1$, you observe a sample of $N(\mu_1, \sigma^2)$. Find the Neyman-Pearson test for false alarm α, i.e. $\Pr(\hat{X} = 1|X = 0) \leq \alpha$.

Problem 5. Problem 1 of Chapter 3, page 57.

Problem 6. Problem 2 of Chapter 3, page 57.