UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS126: Probability in EECS

Problem Set 12
Fall 2013

Issued: Wednesday, November 27, 2013 **Due:** In class Thursday, December 5

Problem 1. Consider a routing network with 3 nodes. The source node s, the destination node d, and a relay node r. There is a direct path from s to d with travel time 20. The travel time from s to r is 15. There are 2 paths from r to d. One of them uniformly distributed between 4 and 8, and the other one uniformly distributed between 3 and 11.

(a) If you want to do pre-planning, which path should be chosen to go from s to d?

(b) If the travel times from r to d are revealed at r which path should be chosen?

Problem 2. Consider 2 queues in parallel in discrete time with Bernoulli arrival processes of rates λ_1 and λ_2, and geometric service rates of μ_1 and μ_2, respectively. There is only one server that can serve either queue 1 and queue 2 at each time. Consider the scheduling policy that is at time n serve queue 1 if $\mu_1 Q_1(n) > \mu_2 Q_2(n)$, and serve queue 2 otherwise, where $Q_1(n)$ and $Q_2(n)$ are queue-lengths of the queues at time n. Use the Lyapunov function $V(Q_1(n), Q_2(n)) = Q_1^2(n) + Q_2^2(n)$ to show that the queues are stable if $\lambda_1/\mu_1 + \lambda_2/\mu_2 < 1$. This scheduling policy is known as Max-weight or Back-pressure policy.

Problem 3. Consider a simple problem of routing traffic in queueing network. There are 2 queues in parallel each of them equipped with one server. Let's $X(n)$ and $Y(n)$ denote the queue-lengths of the queues at time n. Both of the queues have exponential service with rate rate μ. Customers arrive to the network according to Poisson process with rate λ and decide which queue to join. Time slots are defined based on every "event" happening in the network. There are 3 possible events: 1) Arrival, 2) Departure from queue 1, 3) Departure from queue 2. With each event happening, the time of the system increases by 1. The objective is to minimize the expected sum of discounted queue-lengths:

$$E[\sum_n \beta^n (X(n) + Y(n))],$$

where $0 < \beta < 1$.

(a) Argue that the dynamic programming equations are:

$$V_{n+1}(x, y) = x + y + \beta\Big(\frac{\lambda}{\lambda + 2\mu} \min\{V_n(x + 1, y), V_n(x, y + 1)\} + \frac{\mu}{\lambda + 2\mu} V_n((x - 1)^+, y) + \frac{\mu}{\lambda + 2\mu} V_n(x, (y - 1)^+)\Big),$$

where $x^+ = \max(0, x)$.

(b) Use induction to show that the optimal strategy is joining the shortest queue. [Hint: Your induction hypothesis should be $V(x, y) \le V(a, b)$ if $x + y \le a + b$ and $|x - y| \le |a - b|$.]

Problem 4. Consider a single queue in discrete time with Bernoulli arrival process of rate λ. There is one server dedicated to the queue with service rate is μ. You can decide to allocate another server to the queue that increases the rate to $\mu(1 + \alpha)$ for some $\alpha > 0$. However, using the additional server has some cost. Suppose the overall cost at time n is $X(n) + H(n)$, where $X(n)$ is the queue-length and $H(n)$ is the indicator that the additional server is used at time n. (It is 1 if the additional server is used, and 0 otherwise.) You want to minimize the cost over a finite horizon.

$$\sum_{n=0}^{N} E(X(n) + H(n))$$

(a) Write the dynamic programming equations.

(b) Solve the DPE with MATLAB for $\alpha = 0.5$ and $N = 50$.