UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 9
Fall 2007

Issued: Thursday, November 1, 2007 **Due:** Friday, November 9, 2007

Reading: Bertsekas & Tsitsiklis, Chapter 4, §5.1

Chapter 4 problems

Problem 9.1

For this problem, we consider the form of the linear least squares estimator of X when two measurements are available. More specifically, find a_1, a_2, and b such that the linear predictor $g(Y_1, Y_2) = a_1 Y_1 + a_2 Y_2 + b$ minimizes

$$\mathbb{E}[(X - g(Y_1, Y_2))^2] \;=\; \mathbb{E}[(X - a_1 Y_1 - a_2 Y_2 - b)^2].$$

To make the problem a little more tractable, assume that Y_1 and Y_2 are uncorrelated (i.e., $\mathbb{E}[Y_1 Y_2] = \mathbb{E}[Y_1]\mathbb{E}[Y_2]$).

Problem 9.2

Bob has gone hiking, and is lost in the forest. In order to try and find a road, he decides on the following *distance/coin-flip* strategy. At time instants $t = 1, 2, 3\ldots$, he chooses a distance uniformly at random between t and $t+1$. Independently of the chosen random distance, he then flips a fair coin; if it comes up heads, he moves the chosen random distance to the right (positive on the real line), and otherwise for a tails toss, he moves the chosen random distance to the left (negative on the real line). Both the random distance and the coin flip are independent random variables for different time instants. Assume that he starts at the origin at time instant $t = 0$.

(a) Let Y_s be Bob's position after repeating his distance/coin-flip strategy for a fixed number of s time instants. Compute its expected value and variance as a function of s.

Now suppose that Bob repeats his distance/coin-flip strategy for a *random number* S of time rounds, after which he stops. Assume that $S \sim \text{Geo}(p)$ has a geometric distribution with parameter p, and let $X \in \mathbb{R}$ be his final position. For any question below, you may feel free to express your answer (if appropriate) in terms of the moments $\mu_i = \mathbb{E}[S^i], i = 1, 2, 3\ldots$.

(b) Suppose that you observe that $S = s$. What is the minimum mean squared error (MMSE) estimator of X given this information?

(c) What is the expected value and variance of his position X?

(d) Now suppose that you observe that Bob finishes at position $X = x$. Given this information, what is the linear least squares estimator (LLSE) of the number of time rounds S that he repeated his distance/coin-flip strategy?

(e) What is the linear least-squares estimate of S based on X^2?

Chapter 5 problems

Problem 9.3

Each of n packages is loaded independently onto either a red truck (with probability p) or onto a green truck (with probability $1-p$). Let R be the total number of items selected for the red truck and let G be the total number of items selected for the green truck.

(a) Determine the PMF, expected value, and variance of the random variable R.

(b) Evaluate the probability that the first item to be loaded ends up being the only one on its truck.

(c) Evaluate the probability that at least one truck ends up with a total of exactly one package.

(d) Evaluate the expected value and the variance of the difference $R-G$.

(e) Assume that $n \geq 2$. Given that both of the first two packages are loaded onto the red truck, find the conditional expectation, variance, and PMF of the random variable R.

Problem 9.4

Fred is giving out samples of dog food. He makes calls door to door, but he leaves a sample (one can) only on those calls for which the door is answered *and* a dog is in residence. On any call the probability of the door being answered is $3/4$, and the probability that any household has a dog is $2/3$. Assume that the events "Door answered" and "A dog lives here" are independent and also that the outcomes of all calls are independent.

(a) Determine the probability that Fred gives away his first sample on his third call.

(b) Given that he has given away exactly four samples on his first eight calls, determine the conditional probability that Fred will give away his fifth sample on his eleventh call.

(c) Determine the probability that he gives away his second sample on his fifth call.

(d) Given that he did not give away his second sample on his second call, determine the conditional probability that he will leave his second sample on his fifth call.

(e) We will say that Fred "needs a new supply" immediately *after* the call on which he gives away his last can. If he starts out with two cans, determine the probability that he completes at least five calls before he needs a new supply.

(f) If he starts out with exactly m cans, determine the expected value and variance of D_m, the number of homes with dogs which he passes up (because of no answer) before he needs a new supply.