UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probability and Random Processes

Problem Set 7
Fall 2007

Issued: Thursday, October 18, 2007 **Due:** Friday, October 26, 2007

Reading: Bertsekas & Tsitsiklis, §4.1–4.4

Problem 7.1
Suppose X and Y are independent and exponentially distributed, each with parameter λ.

(a) Find the CDF and PDF of $Z = X/Y$. Is $E[Z]$ less than, equal to, or greater than 1?

(b) Find the joint density function of Z as defined in (a) and the random variable W, where $W = X + Y$. Are Z and W independent?

Problem 7.2
Consider random variable Z with transform $M_Z(s) = \frac{8-3s}{s^2-6s+8}$.

(a) Find $P(Z \geq 0.5)$.

(b) Find $E[Z]$ by using the probability distribution of Z.

(c) Find $E[Z]$ by using the tranform of Z and without explicity using the probability distribution of Z.

(d) Find $var(Z)$ by using the probability distribution of Z.

(e) Find $var(Z)$ by using the tranform of Z and without explicity using the probability distribution of Z.

Problem 7.3
The number of customers K who shop at a supermarket in a day has the PMF $p_K(k) = \frac{\lambda^k e^{-\lambda}}{k!}$ for $k = 0, 1, 2, \ldots$ and, independent of K, the number L of items purchased by any customer has the PMF $p_L(l) = \frac{\mu^l e^{-\mu}}{l!}$ for $l = 0, 1, 2, \ldots$. Two ways the supermarket can obtain a 10% increase in the expected value of the number of items sold are:

(a) To increase μ by 10%.

(b) To increase λ by 10%.

Which of these changes would lead to the smaller variance of the total items sold per day?

Problem 7.4
X and Y are continuous, independent random variables. The transform of X is given by $M_X(s) = \frac{1}{s}(e^{4s} - e^{3s})$, and the distribution of Y is given by

$$f_Y(y) = \begin{cases} 3c, & \text{for } -2 \leq y \leq -1, \\ c, & \text{for } 0 \leq y \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the numerical value of the constant c.

(b) Compute the transform $M_Y(s)$.

(c) Find the mean and variance of Y.

(d) Find the transform $M_W(s)$, where $W = \alpha X + \beta Y + \gamma$.

(e) Determine the PDF of W for the case where $\alpha = 1$, $\beta = 1$, and $\gamma = 0$.

Problem 7.5

Steve is trying to decide how to invest his wealth in the stock market. He decides to use a probabilistic model for the share price changes. He believes that, at the end of the day, the change of price Z_i of a share of a particular company i is the sum of two components: X_i, due solely to the company, and the other Y due to investors' jitter. Assume that $Y \sim N(0, 1)$ is standard normal and independent of X_i. Find the PDF of Z_i under the following circumstances in part a) to c),

(a) X_1 is Gaussian with a mean of 1 dollar and variance equal to 4.

(b) X_2 is equal to -1 dollars with probability 0.5, and 3 dollars with probability 0.5.

(c) X_3 is uniformly distributed between -2.5 dollars and 4.5 dollars (No closed form expression is necessary.)

(d) Being risk averse, Steve now decides to invest only in the first two companies. He uniformly chooses a portion V of his wealth to invest in company 1 (V is uniform between 0 and 1.) Assuming that a share of company 1 or 2 costs 100 dollars, what is the expected value of the relative increase/decrease of his wealth?

Problem 7.6

Oscar is an electrical engineer, and he is equally likely to work between zero and one hundred hours each week (i.e., the time he works is uniformly distributed between zero and one hundred). He gets paid one dollar an hour.

If Oscar works more than fifty hours during a week, there is a probability of 1/2 that he will actually be paid overtime, which means he will receive two dollars an hour for each hour he works longer than fifty hours. Otherwise, he will just get his normal pay for all of his hours that week. Independently of receiving overtime pay, if Oscar works more than seventy five hours in a week, there is a probability of 1/2 that he will receive a one hundred dollar bonus, in addition to whatever else he earns.

Determine the expected value and variance of Oscar's weekly salary.

Problem 7.7

The Kelly strategy Consider a gambler who at each gamble either wins or loses his bet with probabilities p and $1 - p$, independently of earlier gambles. When $p > 1/2$, a popular gambling system, known as the Kelly strategy, is to always bet the fraction $2p - 1$ of the current fortune. Assuming $p > 1/2$, compute the expected fortune after n gambles of a gambler who starts with x units and employs the Kelly strategy.