UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 4
Fall 2007

Issued: Thursday, September 20, 2007 **Due:** Friday, September 28, 2007

Reading: Bertsekas & Tsitsiklis, §2.1—2.5

Problem 4.1
Each of n files is saved independently onto either hard drive A (with probability p) or hard drive B (with probability $1-p$). Let A be the total number of files selected for drive A and let B be the total number of files selected for drive B.

(a) Determine the PMF, expected value, and variance for random variable A.

(b) Evaluate the probability that the first file to be saved ends up being the only one on its drive.

(c) Evaluate the probability that at least one drive ends up with a total of exactly one file.

(d) Evaluate the expectation and the variance for the difference, $D = A - B$.

Problem 4.2
The probability that any particular bulb will burn out during its K^{th} month of use is given by the PMF for K,

$$p_K(k) = \frac{1}{5}(\frac{4}{5})^{k-1}, k = 1, 2, 3, \ldots$$

Four bulbs are life-tested simultaneously. Determine the probability that

(a) None of the four bulbs fails during its first month of use.

(b) Exactly two bulbs have failed by the end of the third month.

(c) Exactly one bulb fails during each of the first three months.

(d) Exactly one bulb has failed by the end of the second month, and exactly two bulbs are still working at the start of the fifth month.

Problem 4.3
A particular circuit works if all ten of its component devices work. Each circuit is tested before leaving the factory. Each working circuit can be sold for k dollars, but each nonworking circuit is worthless and must be thrown away. Each circuit can be built with either *ordinary* devices or *ultra-reliable* devices. An ordinary device has a failure probability of $q = 0.1$ while an ultra-reliable device has a failure probability of $q/2$, independent of any other device. However, each ordinary device costs \$1 whereas an ultra-reliable device costs \$3.

Should you build your circuit with ordinary devices or ultra-reliable devices in order to maximize your expected profit $E[R]$? Keep in mind that your answer will depend on k.

Problem 4.4
Professor May B. Right often has her science facts wrong, and answers each of her students' questions incorrectly with probability 1/4, independently of other questions. In each lecture May is asked either 1 or 2 questions with equal probability.

(a) What is the probability that May gives wrong answers to all the questions she gets in a given lecture?

(b) Given that May gave wrong answers to all the questions she got in a given lecture, what is the probability that she got two questions?

(c) Let X and Y be random variables representing the number of questions May gets and the number of questions she answers correctly in a lecture, respectively. What is the mean and variance of X and the mean and the variance of Y?

(d) Give a neatly labeled sketch of the joint PMF $p_{X,Y}(x, y)$.

(e) Let $Z = X + 2Y$. What is the expectation and variance of Z?

Problem 4.5
Chuck will go shopping for probability books for K hours. Here, K is a random variable and is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops. We are told that $p_{N|K}(n \mid k) = \frac{1}{k}$ for $n = 1, \ldots, k$.

(a) Find the joint PMF of K and N.

(b) Find the marginal PMF of N.

(c) Find the conditional PMF of K given that $N = 2$.

(d) We are now told that he bought at least 2 but no more than 3 books. Find the conditional mean and variance of K, given this piece of information.

Problem 4.6
Every package of Bobak's favorite ramen noodles comes with a plastic figurine of one of the characters in Battlestar Galactica. There are c different character figurines, and each package is equally likely to contain any character. Bobak buys one package of ramen each day, hoping to collect one figurine for each character.

(a) In lecture, we saw that expectation is linear, so that $\mathbb{E}[f(X)] = f(\mathbb{E}[X])$ for any linear function $f(X) = aX + b$ of a random variable X. Prove that the expectation operator is also linear when applied to functions of multiple random variables: i.e., if $f(X_1, \ldots, X_n) = \sum_{i=1}^{n} a_i X_i + b$ for some real numbers $a_1, a_2, \ldots, a_n, b$, then $\mathbb{E}[f(X_1, \ldots, X_n)] = f(\mathbb{E}[X_1], \ldots, \mathbb{E}[X_n])$.

(b) Find the expected number of days which elapse between the acquisitions of the jth new character and the $(j+1)$th new character.

(c) Find the expected number of days which elapse before Bobak has collected all of the characters.