UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 1
Fall 2007

Issued: Thursday, August 30, 2007 **Due:** Friday, September 7, 2007

Reading: Berstsekas & Tsitsiklis, §1.1, §1.2. and §1.3

Problem 1.1
For three tosses of a fair coin, define the sample space. Find the probabilities of the following events:

(a) the sequence HHH?

(b) the sequence HTH?

(c) seeing two heads and one tail?

(d) the outcome "More heads than tails"?

Problem 1.2
In this exercise, we prove Bonferroni's inequality:

(a) Show that for any two events A and B, we have

$$\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1.$$

(b) Generalize to the case of n events $A_1, A_2, \ldots, A_n$ by showing that

$$\mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_n) \geq \mathbb{P}(A_1) + \mathbb{P}(A_2) + \cdots + \mathbb{P}(A_n) - (n-1).$$

Hint: To generalize in part (b), use de Morgan's law.

Problem 1.3
Alice and Emily each choose at random a number between zero and one according to the uniform probability law. Consider the following events:

$$\begin{aligned}
A &= \{\text{The magnitude of the difference of the two numbers is greater than } 1/3.\} \\
B &= \{\text{At least one of the numbers is greater than } 2/3.\} \\
C &= \{\text{The sum of the two numbers is } 1.\} \\
D &= \{\text{Alice's number is greater than } 2/3.\}
\end{aligned}$$

Find the following probabilities: $\mathbb{P}(A)$, $\mathbb{P}(B)$, $\mathbb{P}(A \cap B)$, $\mathbb{P}(C)$, $\mathbb{P}(D)$, $\mathbb{P}(A \cap D)$.

Problem 1.4
Given two events A and B on a common sample space, give expressions for the following probabilities in terms of $\mathbb{P}(A)$, $\mathbb{P}(B)$ and $\mathbb{P}(A \cap B)$.

(a) The probability that *at least one* of A or B occurs?

(b) The probability that *exactly one* of A and B occurs?

Which one of these probabilities is equal to $\mathbb{P}\Big((A \cap B^c) \cup (A^c \cap B)\Big)$?

Problem 1.5

Barbara has a peculiar pair of four-sided dice. When she rolls the dice, the probability of any particular outcome (pair of numbers) is proportional to the product of the outcomes from each die. All outcomes that result in a particular product are equally likely.

(a) What is the probability of the product being even?

(b) What is the probability of Barbara rolling a 2 and a 3?

Problem 1.6

Mike and John are playing a friendly game of darts where the dart board is a disk of a radius of 10 inches. Whenever a dart falls within 1 inch of the center (the bullseye), then 50 points are scored. If the point of impact is between 1 and 3 inches from the center, then 30 points are scored, if it is at a distance of 3 to 5 inches, then 20 points are scored and if it is further than 5 inches, then 10 points are scored. Assume that both players are skilled enough to be able to throw the dart within the boundaries of the board.

Mike can place the dart uniformly on the board (i.e. the probability of the dart falling in a given region is proportional to its area.)

(a) What is the probability that Mike scores 50 points on one throw?

(b) What is the probability of him scoring 30 points?

(c) John is right handed and is twice more likely to throw in the right half of the board than in the left half. Across each half, the dart falls uniformly in that region. Answer the previous questions for John's throw.