UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probability and Random Processes

Discussion Notes: Week 4
Fall 2007

Reading: Berstsekas & Tsitsiklis, §2.1 – §2.5

Key Stuff to Remember:

- **Expectation:**

$$\mathbb{E}[X] = \sum_x x p_X(x)$$

- **Variance:**

$$\begin{aligned}\operatorname{var}(X) &= \sum_x (x - \mathbb{E}[X])^2 p_X(x) \\ &= \mathbb{E}\left[X^2\right] - \mathbb{E}[X]^2\end{aligned}$$

- **Joint PMF Marginalization:**

$$p_X(x) = \sum_y p_{X,Y}(x,y)$$

- **Distributions to Remember:** Bernoulli, Binomial, Geometric, Poisson

Problem 4.1
(Bertsekas 2.3) Fischer and Spassky play a chess match in which the first player to win a game wins the match. After 10 successive draws, the match is declared drawn. Each game is won by Fischer with probability 0.4, is won by Spassky with probability 0.3, and is a draw with probability 0.3, independently of previous games.

(a) What is the probability that Fischer wins the match?

(b) What is the PMF of the duration of the match?

Problem 4.2
(Bertsekas 2.14) Let X be a random variable that takes values from 0 to 9 with equal probability 1/10.

(a) Find the PMF of the random variable $Y = X \mod (3)$.

(b) Find the PMF of the random variable $Y = 5 \mod (X+1)$.

Problem 4.3
(Bertsekas 2.26) A class of n students takes a test consisting of m questions. Suppose that student i submitted answers to the first m_i questions.

(a) The grader randomly picks one answer, call it (I, J), where I is the student ID number (taking values $1, \ldots, n$) and J is the question number (taking values $1, \ldots, m$). Assume that all answers are equally likely to be picked. Calculate the joint and the marginal PMFs of I and J.

(b) Assume that an answer to question j, if submitted by student i, is correct with probability p_{ij}. Each answer gets a points if it is correct and gets b points otherwise. Calculate the expected value of the score of student i.

Problem 4.4
(Bertsekas 2.21) You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

Problem 4.5
A group of entrepreneurs just purchased troubled Air Stanford. Air Stanford currently only offers service to Reykjavik and Auckland. Because they are so disorganized, flights occur in a random manner and their planes often crash. The probability that a flight to Reykjavik or Auckland crashes is 1/5 and 1/10, respectively. Any particular flight goes to Reykjavik with probaility 2/3 and Auckland with probability 1/3.

(a) What is the probability that a randomly chosen flight crashes?

(b) What is the expected number of flights before the first crash?

(c) What is the expected number of flights that occur before the first crash and after 3 non-crash flights?

(d) Air Stanford has 1000 flights per year. What is the probability that they have 1 or fewer crashes in a year.

(e) Suppose that the entrepreneurs discover that if 100 new mechanics are hired, the probability of a safe flight on any particular Air Stanford flight will be 0.9999. Using the Poisson approximation (see p.79 in the textbook), what is the probability that all 1000 flights in a given year arrive safely at their destination?

Problem 4.6
Your EE126 class has 250 undergraduate students and 50 graduate students. The probability of an undergraduate (or graduate) student getting an A is 1/3 (or 1/2, respectively). Let X be the number of students to get an A in your class.

(a) Find the PMF of X.

(b) Calculate $\mathbb{E}[X]$ using the total expectation theorem, rather than the PMF of X.

(c) Calculate $\mathbb{E}[X]$ and $\text{var}(X)$ by viewing X as a sum of random variables, whose statistics are easily calculated.