UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probability and Random Processes

Discussion Notes: Week 10
Fall 2007

Reading: Berstsekas & Tsitsiklis, Chapter 4 and $ 5.1

Key Stuff to Remember:

- **Binomial RV S with parameters p and n:**

$$p_S(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad \mathbb{E}(S) = np, \quad var(S) = np(1-p)$$

- **Geometric RV T with parameter p**

$$p_T(t) = (1-p)^{t-1} p, \quad \mathbb{E}(T) = 1/p$$

- **k-th arrival time**

$$Y_k = T_1 + T_2 + \cdots T_k, \quad T_i : \quad Geo(p)$$
$$E(Y_k) = k/p, \quad var(Y_k) = k(1-p)/p^2$$
$$p_{Y_k}(t) = \binom{t-1}{k-1} p^k (1-p)^{t-k}$$

Problem 10.1
The probability that Iwana Passe fails any quiz is $\frac{1}{4}$. Iwana's performance on each quiz is independent of her performance on all other quizzes.

(a) Determine the probability that Iwana fails exactly two of the next six quizzes.

(b) Find the expected number of quizzes that Iwana will pass before she has failed three times.

(c) Find the probability that the second and third time Iwana fails a quiz will occur when she takes her 8th and 9th quizzes, respectively.

(d) Determine the probability that Iwana fails two quizzes in a row before she passes two quizzes in a row. Equivalently, her first time two-straight-fail happens before any two-straight-pass.

Problem 10.2
For each night, the probability of a robbery attempt at the local warehouse is $\frac{1}{5}$. A robbery attempt is successful with probability $\frac{3}{4}$, independent of the night. After any particular SUCCESSFUL robbery, the robber celebrates by taking off either the next 2 or 4 nights (with equal probability), during which time there will be no robbery attempts. After that, the robber returns to his original routine.

(a) Let K be the number of robbery attempts up to (and including) the first <u>successful</u> robbery. Find the PMF of K.

(b) Let D be the number of days until (and including) the second <u>successful</u> robbery, including the days of celebration after the first robbery. Find the PMF of D, or its transform (whichever you find more convenient).

During a successful robbery, the robber steals a random number of candy bars, which is 1, 2, or 3, with equal probabilities. This number is independent for each successful robbery and independent of everything else (no candy bars are stolen in unsuccessful robberies).

(c) Let W be the number of candy bars collected in two <u>successful</u> robberies. Plot the PMF of W.

(d) Let T be the number of candy bars collected in ten robbery <u>attempts</u> (whether successful or not). Find the PMF of T, or its transform, whichever is easier. Find the expectation and the variance of T.