UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Discussion Notes: Week 1

Fall 2007

Reading: Berstsekas & Tsitsiklis, §1.1, §1.2. and §1.3

Problem 1.1

Which of the following are identically true?

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $(A \cup B) \cap C = A \cup (B \cap C)$

(d) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Problem 1.2

Let $\mathbb{P}(A) = 0.9$ and $\mathbb{P}(B) = 0.8$. Show that $\mathbb{P}(A \cap B) \geq 0.7$

Problem 1.3

Let A and B be events with probabilities $\mathbb{P}(A) = \frac{3}{4}$ and $\mathbb{P}(B) = \frac{1}{3}$. Show that $\frac{1}{12} \leq \mathbb{P}(A \cap B) \leq \frac{1}{3}$, and give examples to show that both extremes are possible. Find corresponding bounds for $\mathbb{P}(A \cup B)$.

Problem 1.4

(Bertsekas 1.2.6) A six-sided die is loaded in such a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than four.

Problem 1.5

(Birthday Problem) *Note: this problem requires use of basic counting principles which will be covered later in the course.*

With n people in a room:

(a) What is the probability that at least two people have the same birthday? *Hint: assume a uniform law and count the size of an appropriate event*

(b) Calculate the probability for n = 65 (enrollment in class right now).

(c) How large do we need n to have the probability in part (a) at least 0.5?

Problem 1.6

(Union Bound / Boole's Inequality) Prove the following:

$$\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \mathbb{P}(A_i) \tag{1}$$

Hint: use induction.