Problem 1.1 *(36pts) True or False. Prove or show a counterexample:*

a. *12pts. If X, Y are non-negative random variables with $E[X] > E[Y]$, then $E[X^2] > E[Y^2]$.*

b. *12pts. If $P(B|A) > P(B|A^c)$, then $P(A|B) \geq P(A)$*

c. *12pts If X is a random variable (continuous or discrete) taking values only in the range $[-1, +1]$, then the variance of X must be less than 1.*

Problem 1.2 *(52pts) Bill has $n > 0$ different stocks in his portfolio. On any particular day, he believes that a stock in his portfolio will ether go up with probability $p > 0$ or down with probability $(1 - p)$ independently of all other stocks (the probability of its staying unchanged is zero).*

a. *12pts. If the experiment is repeated independently every day, how often would the event "more of the stocks held by Bill are up than down" occur? Explain.*

b. *12pts. The portfolio can be divided in three parts: A third of the stocks will either double in price or get divided by 2, a third will triple or get divided by 3, and a third would quadruple or get divided by 4 (assume that n is a multiple of 6; $n = 6m$)*

   What is the expected value of money Bill holds after one day if one dollar is invested equally at the beginning of the day? (i.e. $1/n$ dollars are invested in each of the n stocks initially)

c. *14pts If $n/2$ of the stocks were up on a particular day, what is the probability that they included all the ones that get quadrupled?*

d. *14pts Assume now that the stock market changes in the following manner: For the first day of the year, all stocks have a probability p of going up and $(1 - p)$ of going down. If a stock is down one day, the probability that it will be up the following day is $2p$. If instead the stock was up then it will be up the following day still with probability p.*

   Bill decides the following : for the stocks that lose money on one day, he will invest twice as much (compared to gainers) on the second day.

   On the third day, the stock of Energy Efficient Cooling Systems (EECS) went up. What's the probability that Bill invested more money in EECS on that day?

Problem 1.3 *Every day, Ramzi arrives first to work and parks his car at the edge of the parking lot that consists of n parking spaces in a row. Danielle arrives later and parks her car as close to Ramzi's as possible (Note that answers given as summations are acceptable.)*

a. 16pts. *If k people (excluding Ramzi) parked their cars randomly (in an uniform manner) in the available spots before Danielle arrived, let X be the random variable representing how many spots separate Danielle's and Ramzi's cars (X is zero when the cars are next to each other.) Find the PMF of X.*

b. 15pts. *Denote by p_i the PMF of X at i. If the same experiment is repeated independently every day, find the expected value of days between the times when two spots or less separate Danielle's and Ramzi's cars (Give your answer in terms of the p_i's.)*

c. 15pts *Let Y be the average number of spots separating the two cars over the period of m days, i.e.*

$$Y = \frac{1}{m}\sum_{j=1}^{m} X_j$$

Where X_j is the random variable representing how many spots separate Danielle's and Ramzi's cars on day j.

Find the variance of Y (Give your answer in terms of the p_i's.)