Spring 2005: EECS126 Midterm 1 on March 1, 2005

No Collaboration Permitted. One sheet of notes is permitted. Turn in with your exam.

Be clear and precise in your answers

Write your name and student ID number on every sheet.

Come to the front if you have a question.

Problem 1.1 *(36pts) True or False. Prove or show a counterexample:*

a. 12pts. If A, B, C are independent events, then A is also independent of $B \cup C$.

b. 12pts. If $Var[X+Y] = Var[X] + Var[Y]$, then the random variables X and Y are independent.

More space:

c. *12pts* *Let X be a continuous zero-mean random variable with density $f_X(x)$. The normalization condition for probability implies that $0 \leq f_X(x) \leq 1$ for every x.*

Problem 1.2 *(45pts) You just found out that your manufacturing facility is randomly introducing subtle defects into your chips with probability p.*

If a defective chip makes it to the customer, he will be irate and it will cost you 100 dollars to compensate him.

Throwing out a chip costs 10 dollars, but it guarantees that the customer will not get a defective chip.

The most reliable test for the defect involves looking at a chip by a human technician under a microscope and costs 1000 dollars per chip, so it is never worth doing. However, you do have a cheap test costing only 1 dollar that can flag some of the defects. If the the chip is defective, it will give the right answer with probability q. Unfortunately, even if the chip is not defective, it might incorrectly say that it is defective with probability $\frac{q}{5}$.

a. *5pts. What is the expected cost if you just decide to throw out the chip without even checking it?*

b. *10pts. What is your expected cost under the strategy of simply ignoring the potential defects and just shipping the chip to the customer?*

c. *15pts. What is your expected cost if you decide to cheaply test the chip and throw it out only if the test comes back positive for the defect?*

d. *15pts* *As a function of* p, q*, when is it best to apply strategies (a), (b), or (c)? Interpret your answer.*

Problem 1.3 *(40pts) Simplified Hot Potato Routing*

You are analyzing a simple decentralized file sharing network. Each node treats incoming requests like a "hot potato" and just randomly passes them on to other nodes in the network, unless it has the file itself — in which case it serves the request and does not bounce it anywhere.

Consider the simple "star" network. Your node is at the center and you have some neighbors. Whenever you receive a request that you can not serve, you bounce it to one of your neighbors uniformly at random. If that node can not serve it, they bounce it back to you and the cycle repeats with independent random choices each time.

a. *10pts. Suppose that you only have 2 neighbors.*

   Suppose you receive a request for a file that you do not have, but your right neighbor does. Let X be the random variable counting how many bounces the request encounters before it is served. Derive the PMF for X.

b. *10pts. Now suppose that you have three distinct neighbors. Only one of them has the file of interest. What is the PMF for X in this case and what is its expected value?*

c. 10pts. *Suppose that one of your three neighbors is secretly a cracked node that will report any "subversive" request it observes to the authorities. This cracked node does not host any subversive files itself. One of the other neighbors (not the cracked node) has the subversive file. A request for a subversive file comes in to you, what is the probability that it will end up being reported to the authorities? Does the answer depend on how many neighbors you have?*

d. 10pts. Suppose now that you have n neighbors. Two of them have the file that is requested. Let Y be the number of distinct neighbors that get the request at least once before it is served. What is the expected value for Y?