EECS 126: Probability and Random Processes

Discussion 8

Note: Please work on the problems before the discussion session.

Problem 2. Let X be a random variable such that

$$M(s) = a + be^{2s} + ce^{4s}, \quad \mathbf{E}[X] = 3, \quad \text{var}(X) = 2.$$

Find a, b, and c, and the PMF of X.

Problem 14. Let X_1, and X_2 be independent random variables. Use the moment-generating properties of transforms to verify that $\text{var}(X_1 + X_2) = \text{var}(X_1) + \text{var}(X_2)$.

Problem 6. The transform and the mean associated with a discrete random variable X are given by

$$M(s) = ae^s + be^{4(e^s-1)}, \qquad \mathbf{E}[X] = 3.$$

Find:

(a) The scalar parameters a and b.

(b) $p_X(1)$, $\mathbf{E}[X^2]$, and $\mathbf{E}[2^X]$.

(e) $\mathbf{P}(X + Y = 2)$, where Y is a random variable that is independent of X and is identically distributed with X.

Problem 8. Suppose that

$$M_X(s) = \frac{6 - 3s}{2(1 - s)(3 - s)}.$$

Find the PDF of the associated random variable.