

EECS 126: Probability and Random Processes

Discussion 7

Note: Please work on the problems before the discussion session.

Problem 8. A signal of amplitude $s = 2$ is transmitted from a satellite but is corrupted by noise, and the received signal is $Z = s + W$, where W is noise. When the weather is good, W is normal with zero mean and variance 1. When the weather is bad, W is normal with zero mean and variance 4. Good and bad weather are equally likely. In the absence of any weather information:

- (a) Calculate the PDF of X .

Problem 11. Consider a random variable X with PDF

$$f_X(x) = \begin{cases} 2x/3, & \text{if } 1 < x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$$

and let A be the event $\{X \geq 1.5\}$. Calculate $\mathbf{E}[X]$, $\mathbf{P}(A)$, and $\mathbf{E}[X | A]$.

Problem 14. Alexei is vacationing in Monte Carlo. The amount X (in dollars) he takes to the casino each evening is a random variable with a PDF of the form

$$f_X(x) = \begin{cases} ax, & \text{if } 0 \leq x \leq 40, \\ 0, & \text{otherwise.} \end{cases}$$

At the end of each night, the amount Y that he has when leaving the casino is uniformly distributed between zero and twice the amount that he came with.

- (a) Determine the joint PDF $f_{X,Y}(x, y)$.
- (b) What is the probability that on a given night Alexei makes a positive profit at the casino?
- (c) Find the PDF of Alexei's profit $Y - X$ on a particular night, and also determine its expected value.

Problem 19. Let X be a random variable with PDF f_X . Find the PDF of the random variable $|X|$ in the following three cases.

- (a) X is exponentially distributed with parameter λ .
- (b) X is uniformly distributed in the interval $[-1, 2]$.
- (c) f_X is a general PDF.