EECS 126: Probability and Random Processes

Discussion 4

Note: Please work on the problems before the discussion session.

Problem 2. The annual premium of a special kind of insurance starts at \$1000 and is reduced by 10% after each year where no claim has been filed. The probability that a claim is filed in a given year is 0.05, independently of preceding years. What is the PMF of the total premium paid up to and including the year when the first claim is filed?

Problem 3. Let X be a discrete random variable that is uniformly distributed over the set of integers in the range $[a, b]$, where a and b are integers with $a < 0 < b$. Find the PMF of the random variables $\max\{0, X\}$ and $\min\{0, X\}$.

Problem 9. Imagine a TV game show where each contestant i spins an infinitely calibrated wheel of fortune, which assigns him/her with some real number with a value between 1 and 100. All values are equally likely and the value obtained by each contestant is independent of the value obtained by any other contestant.

(a) Find $\mathbf{P}(X_1 < X_2)$.

(b) Find $\mathbf{P}(X_1 < X_2, X_1 < X_3)$.

(c) Let N be the integer-valued random variable whose value is the index of the first contestant who is assigned a smaller number than contestant 1. As an illustration, if contestant 1 obtains a smaller value than contestants 2 and 3 but contestant 4 has a smaller value than contestant 1 ($X_4 < X_1$), then $N = 4$. Find $\mathbf{P}(N > n)$ as a function of n.

(d) Find $\mathbf{E}[N]$, assuming an infinite number of contestants.

Problem 11. Let $X_1, \ldots, X_n$ be independent, identically distributed random variables with common mean and variance. Find the values of c and d that will make the following formula true:

$$\mathbf{E}\big[(X_1 + \cdots + X_n)^2\big] = c\mathbf{E}[X_1]^2 + d\big(\mathbf{E}[X_1]\big)^2.$$